A Markov Model of Safety Message Broadcasting for Vehicular Networks

Niloofer Toorchi, Mahmoud Ahmadian Attari
Electrical Engineering Department
K. N. Toosi University of Technology, Iran
{niloofer.toorchi@ee, mahmoud@eetd}.kntu.ac.ir

Mohammad Sayad Haghighi, Yang Xiang
School of Information Technology
Deakin University, Australia
{m.sayadhaghighi,yang}@deakin.edu.au

Abstract—Some safety applications in vehicular ad-hoc networks (VANETs) require the dissemination of safety information to all nearby vehicles in a broadcast fashion. Each vehicle should periodically broadcast its state information up to a safety range around itself to avoid likely collisions. This causes a congested channel in dense areas especially in multi-lane roads and leads to significant performance reduction. In this paper, using a Markov model, we analytically derive the percentage of channel utilization as well as the packet transmission rate based on the contention window size, carrier sense range, density of vehicles and packet generation rate. Unlike the previous models, the devised Markov model enables us to derive the probability of packet obsolescence before broadcasting from the probability mass function of service delay. Also, we can evaluate the performance of tracking applications for large and small contention window sizes. The extensive simulations carried out confirm the accuracy of our model.

Keywords—Vehicular Networks; Safety Message; Medium Access Control (MAC); Markov Model

I. INTRODUCTION

Along with the population growth and the dramatic increase in the number of vehicles, the rate of traffic congestion and road accidents has gained momentum. Therefore, transportation management has become a critical issue recently. A large number of projects have been defined to improve the quality of transportation [1]. Future vehicles will be able to exchange information with other vehicles and road-side infrastructure units to prevent dangerous events. This information can be communicated over Dedicated Short Range Communication (DSRC) spectrum in which there is the possibility of high-range broadcasting as well as very low latency communications [2].

To standardize vehicular communications, the Wireless Access in Vehicular Environment (WAVE) protocol stack was defined in accordance with IEEE 802.11p and IEEE1609 standards [3], [4]. The MAC layer uses Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) mechanism to access medium. Some safety applications require that each vehicle periodically broadcasts information such as position, speed, and heading. Therefore, no exchange of RTS and CTS packets happens and only physical carrier sensing is done before broadcasting. Hence, the hidden node problem adversely affects the performance of safety applications. In tracking applications, ensuring that enough information is received by nearby vehicles is of great importance. This issue can be studied from two aspects; one is the probability of safety packet reception which should be kept high so that losing some information does not interrupt correct tracking, and the other one is the quality of propagated information as every packet arriving at the MAC layer queue may not find the chance to be broadcast. The safety packet will be obsolete if a new packet arrives before it is broadcast. Most of the recent researches focus on the former aspect, either analytically or by means of simulation. To the best of our knowledge, the latter one has not been investigated yet. In this paper, we analytically derive some probabilistic indices to evaluate the quality of information dissemination and propose an analytical model to show the relationship between nodes’ MAC layer behavior and the channel utilization. Using this model, we can follow the events in an arbitrary node’s queue and compare the effect of different window sizes on the network behavior.

The rest of the paper is organized as follows: in the next section, we conduct a brief review of the previous related works. In Section III, analytical models are presented to show nodes’ MAC layer behavior in VANETs and the percentage of channel utilization is derived assuming that the vehicles uniformly generate packets containing their state information. Section IV validates the proposed models in the previous section via simulation and investigates the pros and cons of using large and small back-off window sizes. The conclusion is given in Section VI.

II. RELATED WORK

In recent years, research on performance evaluation of broadcast-based services has gained momentum. Moreno et al. [5] outlined a priority access mechanism and found the probability of a broadcast packet being correctly received via simulation. The authors of [6] analytically studied the performance of warning message delivery based on multi-hop communications and derived indices such as the probability that a vehicle is informed. Some researches addressed the problem of controlling the transmission power to improve the performance of safety applications. A number of them such as [7] and [8], benefit from cooperation between vehicles to maintain fairness and prevent unbalanced interference. However, cooperation introduces overheads to the network due to necessity of broadcasting power-related information. Authors of [9, 10] proposed an uncooperative method for power adjustment where each node individually chooses the power based on the feedbacks it receives from the channel.
status. In order to achieve a better tracking performance, the authors proposed that power controlling is done in a way that the channel utilization remains in the interval [0.4,0.8].

In general, there is a dearth of research on the effect of some network parameters such as the back-off window size on the performance of safety applications. A brief study in [11] has shown that bigger back-off window sizes can enhance packet delivery ratio specially in large transmission ranges and high packet generation rates. However, there is still more to study in this field. In this paper, we go through the details of network to mathematically show what changes exactly when the transmission back-off window size varies.

III. ANALYTICAL MODELING OF NETWORK BEHAVIOR FOR TRACKING PURPOSES

A. A Node’s MAC Layer Model

The MAC layer behavior in broadcast-based CSMA is characterized by back-off process of the basic access mechanism [12]. In this section, we model this behavior for uniform packet arrival process with the rate of \( \mu \) packets per second by the multi-dimensional Markov Chain (MC) depicted in Fig. 1. First of all, we enumerate the assumptions which have been made to simplify the analysis without loss of generality. It is assumed that all the nodes have equal carrier sense range (\( L_{cs} \)) and all the lanes have equal vehicle density (\( \sigma \)). Similar to [6, 11], we ignore the negligible effect of vehicles’ movement on network connectivity during the interval that a short message is broadcast. In contrast to what is developed in [13], the contention window size (\( \text{CW} \)) remains constant as there is no retransmission or delivery acknowledgement in broadcasting scenarios. Moreover, in tracking scenarios the unsaturated condition holds as a new packet arrival will render the old one obsolete i.e. the old packet is discarded upon arrival of a new one. Therefore, after broadcasting, the queue becomes empty and the node waits for new packet arrival to start contending for the channel again. When the channel contention starts, it may take some time for the node to broadcast the packet. We define \( A_0 \) to be the event that no new packet arrives in the abovementioned servicing interval. In general, \( A_i \) refers to the arrival of \( i \) packets in the service delay interval which leads to the deletion of \( i \) consecutive packets in the queue. The probability of event \( A_i \) occurrence is denoted by \( P_{0i} \). After occurrence of event \( A_i \), the node waits in the state \( q_i \) for a new packet arrival to trigger the contention process.

To solve the Markov chain, we need to find the values of \( P_{00}, P_{01}, ..., P_{0i} \) as well as the values of \( P_0, P_1, ..., P_i \) which are the probabilities of remaining in the waiting states \( q_0, q_1, ..., q_i \) in each \( T_{\text{vs}} \), respectively. \( T_{\text{vs}} \) is the average transition time from one state to another in the given MC and is equal to \( \frac{\bar{m}_{\text{vs}}}{T_{\text{vs}}} \), where \( \bar{m}_{\text{vs}} \) is the expected number of time slots between two consecutive decreases of the back-off counter (a virtual slot), and \( T_{\text{vs}} \) is a slot time duration. The quantities \( \bar{m}_{\text{vs}}, P_{0j}, P_j \) and \( P_i \), \( 0 \leq j \leq i \), will be calculated in the next subsections. Reaching the state ‘0’ means a transmission has been carried out. Thus we can derive a node’s average probability of transmission in a virtual slot (denoted by \( \tau \)) through finding the probability of being in the state ‘0’ as follows:

\[
\tau = \frac{1}{\Sigma_{j=0}^{\infty} \frac{P_{0j}}{P_j + 1 + \frac{W_0-1}{2}}}
\]

Since in every virtual slot there is only one idle slot (where the node is allowed to decrease the back-off counter or start a transmission), \( \tau \) can also be thought to be the probability of transmission in an idle slot. We refer to the node experiencing an idle slot as an active node.

B. Calculation of \( \bar{m}_{\text{vs}} \)

Precise calculation of \( \bar{m}_{\text{vs}} \) is very difficult since it depends on the mutual interaction of nodes in the network. We propose a simple method to approximately find \( \bar{m}_{\text{vs}} \). Let us define the random variable \( m_{\text{vs}} \) as the number of slots between two consecutive decreases of the back-off counter. \( m_{\text{vs}} = 1 \) corresponds to the event that an idle slot is experienced conditioned on the fact that the previous one has also been idle. We denote the probability of this event by \( P_{\text{idle}} \). Therefore, a node senses start of a transmission with the probability of \( (1 - P_{\text{idle}}) \) and suspends the countdown process after observing an idle slot. Calculation of the length of back-off suspension interval in broadcast-based scenarios is very challenging due to the hidden node phenomenon. If \( n_p \) is the length of safety packets in terms of time slots, \( n = n_p + \frac{\text{DIFS}}{T_s} \) will be the minimum number of slots the surrounding nodes suspend their back-off process as transmissions from hidden nodes may make this interval longer. Let us define the random variable \( s_p \) as \( s_p \in [n, n + 1, n + 2, ..., \infty] \) as the number of slots the node suspends its back-off countdown after a transmission is sensed to be started. Knowing \( P_{\text{idle}} \) and the probability mass function (pmf) of \( s_p \), the pmf of \( m_{\text{vs}} \) is also known and the expected value of \( m_{\text{vs}} \) can be calculated as below:

\[
\bar{m}_{\text{vs}} = P_{\text{idle}} \times 1 + (1 - P_{\text{idle}}) (P_{s_1} \times (n + 1) + P_{s_2} \times (n + 2) + ...)
\]

To calculate \( P_{\text{idle}} \), we need to know the number of active nodes within the carrier sense area (CSA) of the tagged node (a generic node in network that we are calculating quantities from its point of view) when it observes an idle slot, denoted by \( N_d \). Then, using \( \tau \), we can simply derive \( P_{\text{idle}} \) as below:

\[
P_{\text{idle}} = (1 - \tau)^{N_d}
\]
A simple method has been proposed in [10] to calculate \( N_a \) for a single-lane road with uniform distribution of nodes. This method can be easily extended for multi-lane scenarios. For example, for a four-lane road, we can consider the uniform scenario shown in Fig. 2 and use the proposed relations in [10] to derive \( P_{stl}(i) \) as the probability that each of four nodes with the horizontal coordinate \( i \) (\( 1 \leq i \leq k, k = [\sigma L_{cs}] \)) in Fig. 2 is active. It is only required to substitute \( 4 \times i \) for \( i \) in the paper relations and due to symmetry of the considered scenario, we get \( N_a = 3 + 2\Sigma_{i=1}^{k} 4 \cdot P_{stl}(i) \).

To derive the pmf of \( s_p \), it is also needed to know \( N_b \); the expected number of active nodes within \( csa \) of the tagged node (tg) when it is not active. To this end, we consider two cases: the first is when tg senses transmissions from both sides of itself. This approximately leaves no active node in the \( csa \) of the tg. The second case is when it senses transmissions from only one side and hence, some active nodes may exist on the other side. Thus, the occurrence probabilities of these two cases, \( P_{c1} \) and \( P_{c2} \) and the expected number of active nodes in the second case \( (NAN_2) \) must be known. Then we have:

\[
N_b = P_{c1} \times 0 + P_{c2} \times NAN_2
\]  

Let us define \( Q \) to be the expected number of active nodes in the \( csa \) and in a generic slot. The quantity \( (1 - \tau)^n \) shows the probability that no node transmits in \( n \) consecutive slots throughout the \( csa \). Although these \( n \) consecutive slots are not independent, for the sake of problem simplification we assume they are. The results will show that this does not introduce a significant error. Similar to [11], we simplify the calculations through approximating \( (1 - \tau)^n \) by \( e^{-\tau Q n} \).

If \( l \) is the number of lanes, \( N = 2l\sigma L_{cs} \) is the expected number of nodes within each node’s \( csa \) (we do not differentiate between the nodes in different lanes and ignore the distance between the lanes with respect to the length of \( L_{cs} \)). The probability that each of them is active in a slot can be estimated via the recursive relation \( p = e^{-\tau n Lp} \) (i.e. the probability that no node in the \( csa \) has started transmission during the previous \( n \) slots). Obviously we will have \( Q = Np \). One can break the probability of having at least one transmission in \( n \) slots into two terms as follows:

\[
1 - e^{-\tau Q n} = \left( 1 - e^{-\frac{\tau n}{l}} \right) + 2 \left( 1 - e^{-\frac{\tau n}{l}} \right) e^{-\frac{\tau n}{2}}
\]  

The first term is the probability that at least one transmission has occurred in each of the \( r \)'s sides and the second one is the probability that at least one transmission has occurred in only one side (either right or left) during the previous \( n \) slots. Thus, the conditional probabilities \( P_{c1} \) and \( P_{c2} \) are found as:

\[
P_{c1} = \frac{(1-e^{-\frac{\tau n}{l}})^2}{1-e^{-\tau Q n}}, P_{c2} = 2 \frac{(1-e^{-\frac{\tau n}{l}}) e^{-\frac{\tau n}{2}}}{1-e^{-\tau Q n}}
\]

We present an approach to find \( N \) for the four-lane roadway depicted in Fig. 3. It can be similarly extended for multiple lanes. Clearly, the active area depends on the location of closest transmitter in the \( csa \) to the \( tg \) (node \( h_1 \)) and closest transmitter in hidden nodes area (node \( h_2 \)). We define \( x \) and \( y \) to be the random variables representing the distance of \( h_1 \) and \( h_2 \) from the outer boundaries, respectively. The cumulative distribution function of \( x \) and \( y \) can be derived as follows [11]:

\[
P(X \leq x) = e^{-4\sigma(L-cs-x)p_{prn}}, P(Y \leq y) = e^{-4\sigma(L-cs-y)p_{prn}}
\]

From the above equations, the probability density function (pdf) of \( x \) is found as follows:

\[
P(x) = \begin{cases} 
(4\sigma p_{prn})e^{-4\sigma(L-cs-x)p_{prn}} & x \in (0, L_{cs}) \\
e^{-4\sigma L_{cs}p_{prn}} & x = 0
\end{cases}
\]

The pdf of \( y \) is the same as \( P(x) \). However, we need to know the pdf of \( x \) under the condition that at least one transmission has been sensed from one side of the \( tg \) during the previous \( n \) slots (event \( c_2 \)). Hence, we have:

\[
P(x|c_2) = (4\sigma p_{prn})e^{-4\sigma(L-cs-x)p_{prn}}/(1 - e^{-4\sigma L_{cs}p_{prn}})
\]

From Fig. 3, we can find the expected number of nodes in active area via \( P(x|c_2) \) and \( P(y) \) as below:

\[
NAN_2 = \int_{x=0}^{L_{cs}} \int_{y=0}^{L_{cs}} 4\sigma(L_{cs} - x - y) \times P(x|c_2)(4\sigma p_{prn})e^{-4\sigma(L-cs-y)p_{prn}} dx dy
\]

The first term of the above equation is related when there are also transmissions in the hidden nodes area and the second term is related when there is no transmitting hidden node i.e. \( y = 0 \). By substituting (10) and (6) in (4), \( N_b \) will be found.

Having \( N_b \), we can derive the pmf of \( s_p \) using the tree diagram shown in Fig. 5. To this end, we define the crucial slot as the latest slot (among \( n \) slots of a packet) during which
In order to find as the service delay, i.e. the number of time slots elapses from start of the channel contention to the broadcast moment. This variable depends on two other variables; can be found as: The occurrence probabilities of the paths leading to the same value of shows the occurrence probability of the example in Fig. 4 as value), the pmf of is back-off window which has a uniform distribution with the probability of is the maximum of . If we assume that the number of slots between two back-off counter decreases is the constant mean value and accordingly is the number of choices of , is the number of slots between two arrivals of packets. In addition to the abovementioned equation, there is an alternative solution which approximately calculates where . The second term is equal to the probability that at least one transmission is started during the first slot and no transmission is started during the next times convolution of the pmf of . Then we have where and the occurrence probability of the event is written above the branch. We may decompose the probability “1” as follows:

\[
1 = e^{-N_b t} + e^{-N_b t(1-1)} + e^{-N_b t(1-2)} \times \cdots \times (1 - e^{-N_b t}) + \cdots + (1 - e^{-N_b t})
\]

The first term refers to the probability that the start of no transmission is sensed during the first packet which results in . The second term is equal to the probability that at least one transmission is started during the first slot and no transmission is started during the next slots within , i.e. the crucial slot of the first packet is the first one. This event adds one slot to the busy interval. The third term shows the probability that the second slot is crucial which adds two slots to the busy interval and the last term is the probability that during the th slot of the first packet, senses the start of a new transmission (i.e. adding more busy slots). In the next step, we must investigate the crucial slot among the added slots. If is the number of added slots, is the probability that the start of no transmission is sensed during these slots and busy interval ends when . We have shown the total busy intervals at the end of the path in Fig. 5 and the values in the braces are temporal. The term indicates the probability that the crucial slot is the one among these slots which results in adding busy slots that might be branched further in the next steps. The path outlined with dashed arrows in Fig. 5 (which ends to ) shows the occurrence probability of the example in Fig. 4 as . The occurrence probabilities of the paths leading to the same value of must be added together to find the corresponding probability of each . We noticed that after six or seven steps, the resultant does not change much. This point can be used to speed up calculations in practice.

\[
P_{0j} = P\left( \frac{j}{\mu T_s} \leq d < \frac{j+1}{\mu T_s} \right), \quad j = 0, 1, 2, \ldots
\]

where is the number of slots between two arrivals of packets. In addition to the abovementioned equation, there is an alternative solution which approximately calculates . If we assume that the number of slots between two back-off counter decreases is the constant mean value , there remains only the random variable . Therefore, the expected delay to the broadcast moment due to the choice of , will be . If is the number of choices of so that the condition holds, then:

\[
P_{0j} = x_j / W_0, \quad 0 \leq j \leq i
\]

where is the maximum of . We use the approximate solution to have a finite number of waiting states in the Markov model of Fig. 1. In the next section, we will show that the approximate model well chases the precise one.

Following the geometric distribution rules, where is the expected number of virtual slots the model
remains in the state \( a_j \). The queue is empty from the broadcast moment till the next packet arrival moment:

\[
z_j = ((j+1)/\mu T_{vs}) - y_j , \quad 0 \leq j \leq i
\]  

(14)

where \( y_j \) is the expected service delay knowing that \( A_j \), \( 0 \leq j \leq i \) has occurred. To derive \( y_j \), we must average the expected service delays (in virtual scale) due to each \( w_j \) occurrence. For instance, since \( w_0 \) can take the values 0, 1, ..., \( x_0 - 1 \), we have:

\[
y_0 = (1/x_0)(0 + \cdots + x_0 - 1) + 1 = 1 + (x_0 - 1)/2
\]  

(15)

In general, the set \( \{\sum_{k=0}^{j-1} x_k, \ldots, \sum_{k=0}^{i-1} x_k + x_j - 1\} \) contains the values can be taken by \( w_j \). Hence:

\[
y_j = 1 + (\sum_{k=0}^{j-1} x_k + (x_j - 1)/2) , \quad 1 \leq j \leq i
\]  

(16)

The reason for adding 1 to the Equations (15) and (16) is related to the structure of the MC in Fig. 1. Since every transition takes one virtual slot, we have one virtual slot from the state ‘0’ to the state \( a_0 \). Adding 1 to these equations will remove the effect of this delay in (1). Since the calculation of \( P_{0j} \) and \( P_j \) depends on \( \tau \), (1) must be solved numerically.

Knowing \( \tau \) and \( \overline{m}_{vs} \), the rate of packet transmission will be found through \( \lambda = \tau/T_{vs} \).

In practice, an interesting point is that each node can find the value of \( \overline{m}_{vs} \) itself via the clear channel assessment reports and it does not require to know the number of nodes in its csa. If \( P_b \) is the ratio of inactive slots to the total number of slots in an analysis window of sufficient length, then:

\[
\overline{m}_{vs} = 1/(1 - P_b)
\]  

(17)

IV. COMPARISONS AND NUMERICAL RESULTS

In this section, we validate the proposed analytical model and compare its results with those obtained by means of simulations. We developed MATLAB code to simulate a 10km four-lane highway using the parameters \( T_c = 16\mu s \) and \( DIFS = 64\mu s \) and assume that the overall size of transmitted packet is 300B and the data rate is 6Mbps. This results in \( n_p = 25 \) slots. All the nodes perform the basic access mechanism of IEEE 802.11 before broadcasting.

To evaluate the accuracy of the model derived in Subsection B of Section III, we have plotted \( \overline{m}_{vs} \) using the \( \tau \) obtained from simulations and compared it against the simulation results for \( W_0 = 1024 \) in Fig. 6. This figure indicates that given \( \tau \), the theoretically derived pmf can successfully estimate \( \overline{m}_{vs} \) when \( \overline{m}_{vs} < 15 \). However, for more congested channels, the proposed model cannot estimate the value precisely. Part of this is due to the approximations made in the calculation of \( N_b \). This plot also confirms the accuracy of (17).

Fig. 7 demonstrates the service rate (\( \lambda \)) obtained both analytically and by means of simulation. The curves comparison confirms the accuracy of MC. Again, the minor difference is due to inexact estimations of \( \overline{m}_{vs} \) in highly congested areas. It can be seen that for \( \overline{m}_{vs} < 6 \), \( \lambda \) is equal to the packet generation rate \( \mu \). When the channel gets busier, \( \lambda \) decreases more and more. We look into the reason of this phenomenon in Fig. 8 where \( P_{0j}, 0 \leq j \leq 3 \) which have been found (precisely) via (12) and (approximately) via (13), are depicted for different \( \overline{m}_{vs} \) values. This figure shows that for \( \overline{m}_{vs} > 6 \), \( P_{00} \) becomes less and less which means some packets are discarded. The figure also confirms that the approximate curve chases the precise one very well. Since the approximate method converts low probabilities to zero, it leads to a finite number for \( \overline{m}_{vs} \) values has been validated through the comparison with simulation results.

It is clear that \( P_{0j} \) depends on \( \mu, W_0 \) and \( \overline{m}_{vs} \). For small values of \( W_0 \), busier channels are required to make \( P_{0j} \) non-zero. For instance, as it is shown in Fig. 7 and Fig. 8, for \( \mu = 10 \) and \( W_0 = 1024 \), it is expected that \( P_{01} \) remains almost zero as long as \( \overline{m}_{vs} \) is smaller than 6 slots (where \( (W_0 - 1)T_{vs} \) gets to 100 ms). Thus, when \( P_b > 0.83 \), a value of 1024 for \( W_0 \) will decrease the rate of packet transmission, however, for \( W_0 = 128 \), a very congested channel (i.e. \( P_b > 0.97 \) is needed to make \( \lambda \) drop.

The channel utilization has been plotted in Fig. 10 sweeping the nodes’ density for two values of \( W_0 \). This figure indicates that both back-off window sizes result in the same channel utilization when \( P_b < 0.8 \). However, when density increases and the channel gets busier, longer back-off mitigates the congestion of channel through reduction of
packet transmission rate. Although this can help to enhance the packet delivery probability, it comes at the cost of more packet loss in the queues, which is an indicator of low information propagation quality.

Fig. 8 and Fig. 9 show that there is the probability of losing two consecutive packets with $W_0 = 1024$ when $m_{\infty} > 11$ which means that there is a probability that vehicle's information is not broadcast for 300ms. This event may affect the performance of tracking applications. Hence, as long as $P_b < 0.91$, one can expect that the interval between two consecutive broadcasts is either 100ms or 200ms.

The proposed model paves the way for choosing an appropriate back-off window in different network conditions. Since bigger $W_0$ can decrease the probability of collisions caused by concurrent transmissions [11], it is necessary to know the maximum of $W_0$ suitable for each safety application and under different channel conditions. For instance, consider an application with 50 Hz packet generation rate (e.g. pre-crash warning in [2]). To have $P_{90} \approx 1$, concerning the utilization of the channel in that moment, the largest contention window must satisfy the constraint $(W_0 - 1)R_{\text{max}}T_s < 20$ ms. For example, if a channel utilization of $P_b = 0.9$ is reported, $W_0 = 64$ will be the maximum suitable size of the contention window (among the standard sizes). For regular applications which are tolerant to losses of certain level, choosing larger values for $W_0$, so that the probability of intolerable loss of packets remains zero, can help to mitigate the adverse effects of a busy channel in crowded areas as is shown in Fig. 10. Combining this model with power-controlling methods is in the scope of future studies.

V. CONCLUSION

In this paper, we proposed an analytical model to study MAC layer behavior of nodes in multi-lane VANETs. Based on this model, indices such as the probability of obsolescence of a certain number of consecutive packets in the queue and the channel utilization were derived. These can be useful to analyze the effect of large and small contention window sizes on the quality of advertised information under different network conditions. The results show that a large window can mitigate the channel congestion problem in crowded areas by reducing the effective packet transmission rate.

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