Show or explain all work. If you are unable to submit hardcopy by the deadline, an e-mailed scan by the same deadline is acceptable.

Some Formulae:

Friedmann Equation:
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k c^2}{a^2} + \frac{\Lambda c^2}{3}
\]

Fluid Equation:
\[
\dot{\rho} + 3 \frac{\dot{a}}{a} \left( \rho + \frac{P}{c^2} \right) = 0
\]

Acceleration Equation:
\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3P}{c^2} \right) + \frac{\Lambda c^2}{3}
\]

Question 1:

a) What is the observational evidence for the existence of a cosmological constant? Explain how the data suggest that the expansion of the Universe is accelerating.

b) Using the Friedmann and acceleration equations, and assuming a pressureless Universe, show that for the Universe to be static, the Universe must be closed with a positive cosmological constant.

c) For a pressureless Universe with a cosmological constant, show that the deceleration parameter \( q_0 \) can be expressed as:
\[
q_0 = \frac{\Omega_0}{2} - \Omega_\Lambda(t_0)
\]
Some Constants:

Solar Luminosity $L_\odot \approx 3.8 \times 10^{26}$ W  
Solar Mass $M_\odot \approx 2.0 \times 10^{30}$ kg  

1 pc = $3.086 \times 10^{16}$ m  

$G = 6.673 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$

**Question 2:**

Draco is a dwarf spheroidal galaxy in the Local Group, with a luminosity of $L = (1.8 \pm 0.8) \times 10^5 L_\odot$ and half of its total luminosity contained within a sphere of radius $r_h = 120 \pm 12$ pc. We can measure the radial velocities of individual red giant stars in Draco, and for them we find a line-of-sight velocity dispersion of $\sigma_r = 10.5 \pm 2.2$ km s$^{-1}$.

Using the virial theorem to estimate the mass of a self-gravitating system in equilibrium we get the following formula:

$$ M = \frac{\langle v^2 \rangle r_h}{\alpha G} $$

where $\langle v^2 \rangle$ is the three-dimensional mean square velocity, $r_h$ is the half-mass radius of the system, and $\alpha$ is a factor depending on the density profile of the system (typically of order unity).

**a)** Assuming that all the mass in Draco is distributed in the same way as the luminous mass (that is, the stars), what is the total mass of Draco? Use $\alpha = 0.4$, and assume that the velocity dispersion of the galaxy is isotropic, so that the three-dimensional mean square velocity is just three times the one-dimensional velocity dispersion.

**b)** What is Draco’s mass-to-light ratio (in Solar units)?

**c)** What are some of the potential sources of error in your calculations?