Gravity and Tides

Isaac Newton (1643 - 1727)

- Building on the results of Galileo and Kepler
- Adding physics interpretations to the mathematical descriptions of astronomy by Copernicus, Galileo and Kepler

Major achievements:
1. Invented Calculus as a necessary tool to solve mathematical problems related to motion
2. Discovered the three laws of motion
3. Discovered the universal law of mutual gravitation

Velocity and Acceleration

Acceleration (a) is the change of a body’s velocity (v) with time (t):

\[ a = \frac{\Delta v}{\Delta t} \]

Velocity and acceleration are directed quantities (vectors)!

Different cases of acceleration:
1. Acceleration in the conventional sense (i.e. increasing speed)
2. Deceleration (i.e. decreasing speed)
3. Change of the direction of motion (e.g., in circular motion)
Acceleration of Gravity

The acceleration of gravity is independent of the mass (weight) of the falling object!

Iron ball
Wood ball

Difficult to verify on Earth because of air resistance, but astronauts could verify it easily on the moon.

Newton’s Laws of Motion (1)

1. A body continues at rest or in uniform motion in a straight line unless acted upon by some net force.

An astronaut floating in space will continue to float forever in a straight line unless some external force is accelerating him/her.

Newton’s Laws of Motion (2)

2. The acceleration $a$ of a body is inversely proportional to its mass $m$, directly proportional to the net force $F$, and in the same direction as the net force.

$$a = \frac{F}{m}$$

Newton’s Laws of Motion (3)

3. To every action, there is an equal and opposite reaction.

The same force that is accelerating the boy forward is accelerating the skateboard backward.
The Universal Law of Gravity

• Any two bodies are attracting each other through gravitation, with a force proportional to the product of their masses and inversely proportional to the square of their distance:

\[ F = -\frac{GmM}{r^2} \]

(G is the Universal constant of gravity.)

Understanding Orbital Motion

The universal law of gravity allows us to understand orbital motion of planets and moons:

Example:

• Earth and moon attract each other through gravitation.
• Since Earth is much more massive than the moon, the moon’s effect on Earth is small.
• Earth’s gravitational force constantly accelerates the moon towards Earth.
• This acceleration is constantly changing the moon’s direction of motion, holding it on its almost circular orbit.

Orbital Motion (2)

In order to stay on a closed orbit, an object has to be within a certain range of velocities:

Too slow => Object falls back down to Earth

Too fast => Object escapes Earth’s gravity

Geosynchronous Orbits

At a distance of 42,350 km (26,300 miles) from Earth’s center, a satellite orbits with a period of 24 hours.

The satellite orbits eastward, and Earth rotates eastward under the moon’s satellites.

The satellite remains fixed above a spot on Earth’s equator.
Kepler’s Third Law Explained by Newton

Balancing the force (called “centripetal force”) necessary to keep an object in circular motion with the gravitational force → expression equivalent to Kepler’s third law

\[ P_y^2 = a_{AU}^3 \]

The Tides

Caused by the difference of the Moon’s gravitational attraction on the water on Earth

Forces are balanced at the center of the Earth

Weaker gravity from moon causes water to accumulate on the far side

Stronger gravity from moon causes water to accumulate on the near side

→ 2 tidal maxima

→ 12-hour cycle

Spring and Neap Tides

The Sun is also producing tidal effects, about half as strong as the Moon.

• Near Full and New Moon, those two effects add up to cause spring tides.

• Near first and third quarter, the two effects work at a right angle, causing neap tides.

Acceleration of the Moon’s Orbital Motion

Earth’s tidal bulges are slightly tilted in the direction of Earth’s rotation.

Gravitational force pulls the moon slightly forward along its orbit.
Einstein and Relativity

Einstein (1879 – 1955) noticed that Newton’s laws of motion are only correct in the limit of low velocities, that is, much less than the speed of light.

- Theory of Special Relativity
- Also, revised understanding of gravity
- Theory of General Relativity

Two Postulates Leading to Special Relativity (1)

1. Observers can never detect their uniform motion, except relative to other objects.

   This is equivalent to:

   The laws of physics are the same for all observers, no matter what their motion, as long as they are not accelerated.

Two Postulates Leading to Special Relativity (2)

2. The velocity of light, c, is constant and will be the same for all observers, independent of their motion relative to the light source.

 Basics of Special Relativity

The two postulates of special relativity have some strange consequences.

Consider a thought experiment:

Motion of "stationary" observer Assume a light source moving with velocity $v$ relative to a "stationary" observer.

$\begin{align*}
  v' &:= v/c \\
  c &:= \text{velocity of light} \\
  \Delta t' &:= \text{time interval in observer's frame} \\
  \Delta t &:= \text{time interval in source's frame} \\
\end{align*}$

Light source

Seen by an observer moving along with the light source

$\begin{align*}
  v' &:= v/c \\
  c &:= \text{velocity of light} \\
  \Delta t' &:= \text{time interval in observer's frame} \\
\end{align*}$

Seen by the "stationary" observer

$\begin{align*}
  v &:= \text{velocity of source relative to observer} \\
  \Delta t &:= \text{time interval in source's frame} \\
\end{align*}$
Basics of Special Relativity (2)

Now, recall that the speed of light, \( c \), is the same for all observers.

\( \Rightarrow \) The times \( \Delta t \) and \( \Delta t' \) must be different!

Pythagoras’ Theorem gives:

\[
(c\Delta t)^2 = (c\Delta t')^2 + (v\Delta t)^2
\]

or

\[
\Delta t' = \left(\frac{\Delta t}{\gamma}\right)
\]

where \( \gamma = \frac{1}{\sqrt{1 - [v/c]^2}} \)

is the Lorentz factor.

This effect is called **time dilation**.

Other Effects of Special Relativity

- **Length contraction:** Length scales on a rapidly moving object appear shortened

- **Relativistic aberration:** Distortion of angles

- The energy of a body at rest is not 0. Instead, we find

\[
E_0 = mc^2
\]

General Relativity

A new description of gravity

Postulate:

**Equivalence Principle:**

“Observers cannot distinguish locally between inertial forces due to acceleration and uniform gravitational forces due to the presence of massive bodies.”

Another Thought Experiment

Imagine a light source on board a rapidly accelerated space ship:
Thought Experiment (2)

For the accelerated observer, the light ray appears to bend downward!

Now, we can’t distinguish between this inertial effect and the effect of gravitational forces.

Thus, a gravitational force equivalent to the inertial force must also be able to bend light!

Thought Experiment (Conclusion)

This bending of light by the gravitation of massive bodies has indeed been observed:

During total solar eclipses:
The positions of stars apparently close to the sun are shifted away from the position of the sun.

→ New description of gravity as curvature of space-time

Another manifestation of the bending of light: Gravitational lenses

A massive galaxy cluster is bending and focusing the light from a background object.

Other Effects of General Relativity

• Perihelion advance (in particular, of Mercury)

• Gravitational red shift: Light from sources near massive bodies seems shifted towards longer wavelengths (red).