Electromagnetic radiation (chapter 5)

- visible light and x-rays are two different segments of the electromagnetic spectrum
- particles: photons, energy $E$ \(\sim\) eV for visible light \(\sim\) keV for x-rays quantum mechanics
- electromagnetic waves: wavelength $\lambda$ or frequency $f$
- speed of light, $c = 3.00 \times 10^8$ m/s

For waves in general,

\[
\text{Frequency} \times \text{wavelength} = \text{propagation speed}
\]

# of "full waves" passing a point per second

length of each "full wave" = distance travelled per second

For light (i.e. electromagnetic rad)

\[
\lambda F = c
\]

\[
F = 2 \text{ Hz} \quad \frac{1}{2 \text{ s}}
\]

\[
\lambda = 3 \text{ m}
\]

\[
c = 2 \times 10^8 \times 3 \text{ m} = 6 \times 10^8 \text{ m/s}
\]
- photon energy, \( E = h \nu \)

Planck's constant

\[
\begin{align*}
4.2 \times 10^{-15} \text{ eV.s} \\
6.6 \times 10^{-34} \text{ J.s}
\end{align*}
\]

- \( \lambda, \nu \) or \( E \) are interchangeable

- the electromagnetic spectrum

radio, microwaves, infrared, visible, ultraviolet, x-ray, gamma-ray

increasing \( E \) and \( \nu \)

decreasing \( \lambda \)

e.g., green light has \( \nu = 6 \times 10^{14} \text{ Hz} \)

\[
\lambda = \frac{c}{\nu} = \frac{3 \times 10^8 \text{ m/s}}{6 \times 10^{14} \text{ Hz}} = 5 \times 10^{-7} \text{ m} = 500 \text{ nm}
\]

nanometre (\( 10^{-9} \text{ m} \))

and \( E = hf = 4.2 \times 10^{-15} \text{ eV.s} \times 6 \times 10^{14} \text{ Hz} \)

= 2.5 \text{ eV}
Intensity of radiation = energy per unit time falling on unit area

\[ \text{J/s/m}^2 = \text{W/m}^2 \]

or \( \text{W/cm}^2 \) etc

For x-rays, this is directly proportional to ionisations/second in a detector, so use milli-Roentgen/hr, mR/hr as a measure.

- For a compact source,

\[ I \propto \frac{1}{d^2} \]

inverse-square law

intensity ↑ distance

- same energy/s is spread out over 4x the area
  \( \Rightarrow I \text{ 4x lower} \)

- e.g. if an x-ray source at 1m distance gives an exposure rate of 32 mR/hr,
  at 4m it will be 16x less, i.e. 2 mR/hr

astronomical unit

- e.g. sun at 1 A.U. = 1.5x10^11 m has I comparable to 100W light bulb at a few cm
The production of x-rays (chapter 11)

- electron strikes anode and slows down via interactions with nuclei and atomic electrons

  - initial electron kinetic energy ➔ x-rays 1% ➔ characteristic x-rays
  - heat 99% ➔ bremsstrahlung ➔ 80% of diagnostic x-rays

Bremsstrahlung, or "braking radiation" occurs when an energetic electron is deflected by the electric field near an atomic nucleus

- accelerate an electron ➔ radiation (photon emission)
- decelerate, deflect, <->

- e⁻ 90keV ➔ 30keV
- e⁻ 90keV ➔ violet light 80keV
- atomic nucleus, charge Z e

- e⁻ 60keV ➔ radio transmitter

atomic nucleus, charge Ze
- energy of x-ray photon is determined by distance of electron closest approach to the nucleus
  \[ \Rightarrow \text{continuous spectrum} \]

- many more weak deflections than strong deflections
  \[ \Rightarrow \text{more low-energy photons are produced than high-energy photons} \]

- electron leaves the atom
  \[ \Rightarrow \text{max photon energy} = \text{energy of incident electron} \]

- electron will continue to interact with other nuclei it encounters
  \[ \Rightarrow \text{more low energy photons} \]

\[ \text{# of x-rays} \]

\[ \text{unfiltered} \]

\[ \text{Filtered} \]

\[ kVp = 70 \]

\[ \Rightarrow E_{\text{max}} = 70 \text{ keV} \]

\[ \text{typical } E \approx \frac{1}{5} E_{\text{max}} \]

- low energy x-rays are less penetrating
  \[ \Rightarrow \text{Filter out} \]
  - won't pass through patient but would contribute to exposure to radiation
Characteristic x-rays

- produced by an electronic transition in an atom that fills an inner-shell vacancy induced by a projectile electron

![Diagram of electron transition](image)

- tungsten nucleus
  - $W$, $Z = 74$
  - # of protons "atomic number"

- photon produced by electron changing state $\Rightarrow$ x-ray line
  - definite energy, equal to energy difference between $L \rightarrow K$ shells (or $M \rightarrow K$, etc)

- e.g. $W$
  - $L \rightarrow K$ 58 keV
  - $M \rightarrow K$ 67 keV
    - (less often)
    - [differs for different $Z$]

- original $K$-shell electron must be kicked out
  - takes 69.53 keV at least, so incoming $e^-$ must have $E > 70$ keV

- $K$-characteristic x-rays occur at 70 kVp or higher
  - [For tungsten anode]

- at 110-120 kVp, ~15% of beam is $K$-characteristic x-rays
  - also $L$-characteristic x-rays, etc.
  - low energy
Filtration - thin sheets of Al or other metal attached to output port.
- preferentially blocks low E photons

mA - changes the quantity of X-rays but not the spectrum
- changes # of incident electrons, not their energy

Anode material
- determines amount of bremsstrahlung (increases for higher Z) and the energies of the characteristic x-rays
- usually tungsten (Z = 74)
- Molybdenum used for mammography as lower energy x-rays better for soft-tissue imaging
kVp - determines \( E_{\text{max}} \) # of increases x-ray production x-rays for same mA

\[ E_{\text{peak}} \approx \frac{1}{3} E_{\text{max}} \]

changes too

Time-dependence for single phase, fully-rectified

- strong function of voltage
  \[ \Rightarrow \text{strongly peaked} \]
  \[ \Rightarrow \text{quest for low ripple} \]

Circuit waveform
- low ripple is far more efficient
X-ray interactions
- determine imaging properties

x-rays may be absorbed, scattered, or transmitted.

- absorbed
- scattered
- transmitted

attenuation - reduction of original beam

transmitted x-ray

3 photons were attenuated
3 photons contribute to exit radiation

exit radiation - scattered + transmitted rad that leaves the patient

*typically ~99% of the rad incident on a patient is scattered or absorbed
~ 1% is transmitted

- scattering spoils imaging by reducing contrast
Attenuation
- determined by interaction of x-rays with electrons in patient (in principle also with nuclei)
- increased by increased tissue density, thickness and atomic number of material
- reduced for higher energy x-rays (i.e. these have greater penetration) increasing kVp ⇒ decreased attenuation

<table>
<thead>
<tr>
<th>material</th>
<th>ρ (g/cm²)</th>
<th>mean Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>air</td>
<td>0.0013</td>
<td>7.6</td>
</tr>
<tr>
<td>lung</td>
<td>0.32</td>
<td>7.4</td>
</tr>
<tr>
<td>fat</td>
<td>0.91</td>
<td>6.3</td>
</tr>
<tr>
<td>muscle</td>
<td>1.00</td>
<td>7.4</td>
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<tr>
<td>bone</td>
<td>1.90</td>
<td>13.8</td>
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<tr>
<td>iodine</td>
<td>4.90</td>
<td>53</td>
</tr>
<tr>
<td>barium</td>
<td>3.50</td>
<td>56</td>
</tr>
<tr>
<td>lead</td>
<td>11.40</td>
<td>82</td>
</tr>
</tbody>
</table>

- air, iodine + barium often introduced to generate image contrast
The half-value layer is the amount of a certain material required to attenuate an x-ray beam by 1/2.

Note that 2 of these layers \( \Rightarrow \frac{1}{4} \) of the intensity

half-value layer

Types of x-ray interactions
- ordered from low energy to high energy.
  1. Coherent scattering or classical or Thomson scattering
    - x-ray interacts with an entire atom: absorbed atom emits x-ray with same energy in a different direction.
    - important for x-ray energies below 10 keV, so not important
    - no ionisation

x-ray, same energy
Compton scattering
- x-ray photon scatters off an outer-shell electron and kicks it out of the atom

\[ \text{photon has lower energy \rightarrow different direction} \]
\[ \text{nucleus} \]
\[ \text{outer shell} \]

- effect is almost independent of Z
e.g. bone vs muscle
- decreases slowly with increasing energy
  - dominates photoelectric effect
  - for \( E \approx 30 \text{ keV} \)
- scattered photon may well escape the patient
  - reduce image contrast
  - irradiate bystanders

Pair production
For \( E > 1.02 \text{ MeV} \), photon \( \rightarrow e^+ + e^- \) when passing near an atomic nucleus

Photonuclear disintegration
\( E \geq 10 \text{ MeV} \) photons can kick out "pieces" of the atomic nucleus

- Both processes irrelevant for diagnostic x-rays
Photoelectric effect

- X-ray photon is absorbed by an inner-shell electron, which is ejected from the atom \( \Rightarrow \) usually K-shell "photoelectron"
- Photoelectron is stopped by tissue within 1 mm of ejection site
- Characteristic X-ray emitted as inner-shell vacancy is filled.
  - C, N, or O so very low energy and absorbed in the patient
- Incoming x-ray photon \( \rightarrow \) e- ejected
- Characteristic x-ray
- Probability of this process increases with increasing \( Z \)
  - Bone higher than tissue
  - Decreases with increasing energy x-ray
- This effect is what we exploit in X-ray imaging
  - Incident X-ray photon is absorbed
  - Photoelectron stopped within patient
  - Characteristic X-ray photon also stopped