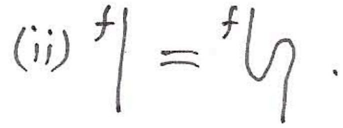
28 Nov 2009

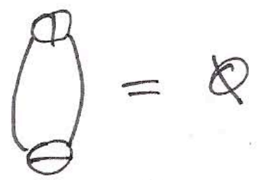
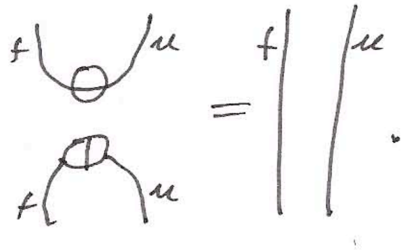
Folklore proposition with string proof

In any bicategory, if $\varepsilon : fu \Rightarrow 1$ is a counit for an adjunction $f \dashv u$ and there exists an invertible 2-cell $\theta : fu \Rightarrow 1$ then ε is invertible.

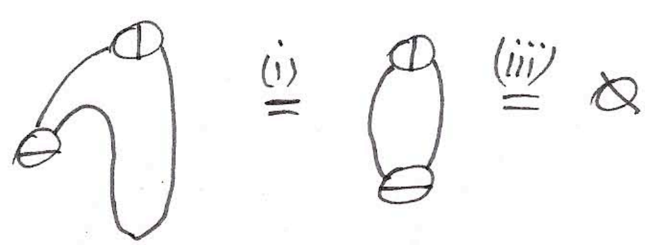
Proof. ε :  and the unit is  so that

(i)  and (ii)  .

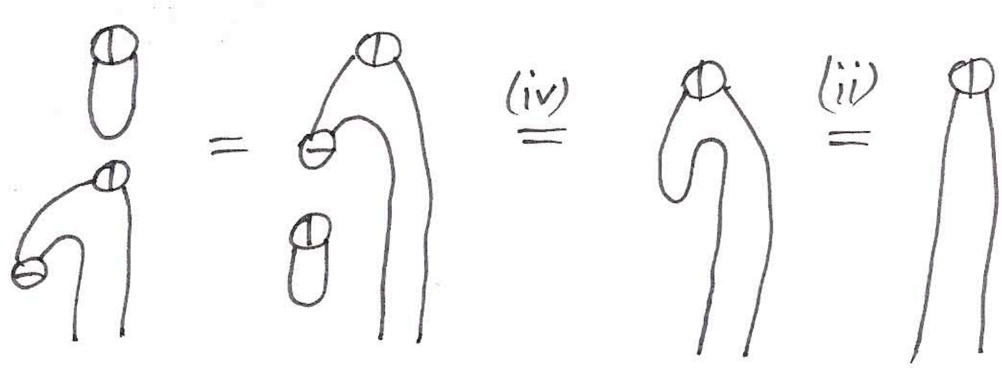
θ :  and inverse  so that

(iii)  and (iv)  .

Put τ :  and calculate

 so that $\varepsilon\tau = 1$;

and



so that $\tau\,\varepsilon\,\theta^{-1} = \theta^{-1}$. That is, $\tau = \varepsilon^{-1}$. $\square$

Rider: (i) & (iii) $\Rightarrow$ ε split epi

(ii) & (iv) $\Rightarrow$ ε split mono