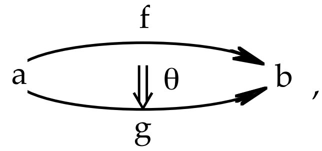
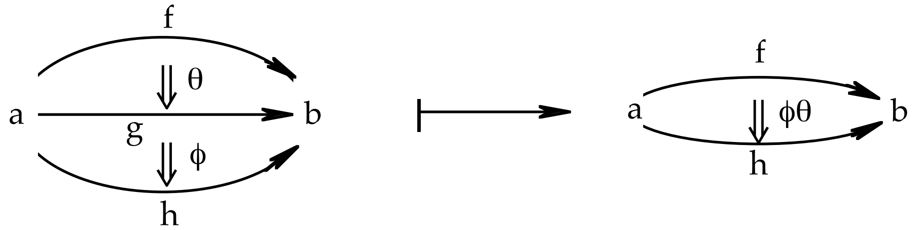
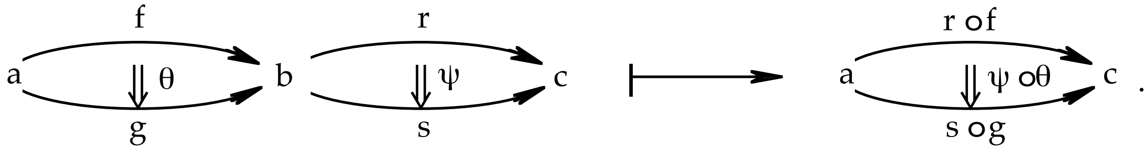
Bicategories and 2-categories

A *2-category* A [Ehr, EK, Gr2, KS, ML] consists of objects a, b, c, . . . , arrows $f : a \dashrightarrow b$, and 2-arrows $\theta : f \Rightarrow g : a \dashrightarrow b$ which can also be displayed thus



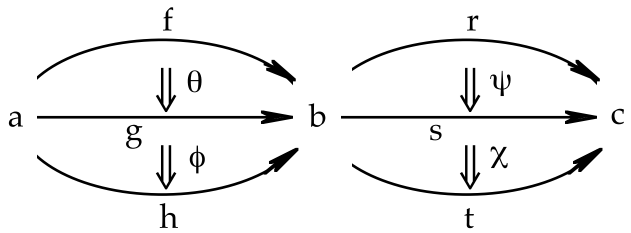
together with vertical and horizontal composition operations





These compositions are required to be associative and unital; moreover, horizontal composition must preserve vertical units and the following *interchange law* is imposed.

$$(\chi\psi) \circ (\phi\theta) = (\chi \circ \phi)(\psi \circ \theta)$$



The basic example of a 2-category is *Cat*: objects are (small) **categories**, arrows are **functors**, and 2-arrows are **natural transformations**. Indeed, the basic "five rules" for composition of natural transformations appeared in the Appendix of [Gt].

There is a weaker notion of 2-category which occurs in practice. A *weak 2-category* or *bicategory* [Bn1] consists of the data and conditions of a 2-category except that the associativity and unital equalities for horizontal composition are replaced by the extra data of invertible natural families of 2-arrows

$$\alpha_{f,r,m} : (m \circ r) \circ f \Rightarrow m \circ (r \circ f), \quad \lambda_f : 1_b \circ f \Rightarrow f, \quad \rho_f : f \circ 1_a \Rightarrow f,$$

called *associativity and unital constraints,* such that the *associativity pentagon* (or *3-cocycle condition*)

$$\alpha_{p,m,r\circ f}\, \alpha_{p\circ m,r,f} = (1_p \circ \alpha_{m,r,f})\, \alpha_{p,m\circ r,f}\, (\alpha_{p,m,r} \circ 1_f)$$

and *unit triangle* (or *normalisation condition*)

$$(1_r \circ \lambda_f)\, \alpha_{f,r,m} = \rho_r \circ 1_f$$

are imposed. In some of the recent literature, bicategories are called 2-categories and 2-categories are called strict 2-categories.

A **monoidal category** $\mathcal{V}$ can be identified with the one-object bicategory $\Sigma\mathcal{V}$ whose arrows are objects of $\mathcal{V}$, whose 2-arrows are the arrows of $\mathcal{V}$, whose horizontal composition is the tensor product of $\mathcal{V}$, and whose vertical composition is the composition of $\mathcal{V}$.

There is a bicategory $\mathcal{M}od$ whose objects are (small) categories and whose arrows are **modules** [St5, St8] (= **profunctors** = **distributors** [Bn2] = **bimodules** [L2]) between categories.

An arrow $f : a \dashrightarrow b$ in a bicategory is called an *equivalence* when there is an arrow $g : b \dashrightarrow a$ such that there are invertible 2-arrows $1_a \Rightarrow g \circ f$ and $f \circ g \Rightarrow 1_b$. A *weak 2-groupoid* is a bicategory in which each 2-arrow is invertible and each arrow is an equivalence. A **2-groupoid** is a 2-category with all arrows and 2-arrows invertible. For each space X, there is a **homotopy 2-groupoid** Π_2X whose objects are the points of X; it contains the information of the **fundamental groupoid** Π_1X and the **homotopy groups** $\pi_2(X,x)$ for each $x\in X$. An early application of 2-categories to homotopy theory occurs in [GZ]. In fact, C. Ehresmann [Ehr] defined *double categories* and *double groupoids* , which generalise 2-categories in that they have two types of arrows (see [KS]), and these also have proved important in homotopy theory [Br].

While many examples occur naturally as bicategories rather than 2-categories, there is a **coherence theorem** asserting that every bicategory is equivalent (in the appropriate sense) to a 2-category [MLP, GPS].

There are several purely categorical motivations for the development of bicategory theory. The first is to study bicategories following the theory of categories but taking account of the 2-dimensionality; this is the spirit of [Gd, Gr2, K2, St3]. A given concept of category theory typically has several generalisations stemming from the fact that equalities between arrows can be replaced by 2-arrow constraints (lax generalization), by invertible 2-arrow constraints (pseudo generalization), or by keeping the equalities; further equalities are required on the constraints. A bicategory can thus be regarded as a pseudo-category, an equivalence as a pseudo-isomorphism, and a **stack** (= "champ" in French) as a pseudo-sheaf. In lax cases there are also choices of direction for the equality-breaking constraints. All this applies to functors: there are *lax functors* (also called *morphisms*) and *pseudo-functors* (also called *homomorphisms*) between bicategories; there are *2-functors* between 2-categories having equality constraints. It also applies to **limits**, **adjunctions**, **Kan extensions**, and the like [Gd, Gr2]. One can use the fact that 2-categories are **categories with homs enriched in** $\mathcal{C}at$; that is, $\mathcal{V}$**-categories** where $\mathcal{V} = \mathcal{C}at$ [EK]. Some laxness is even accounted for in this way: lax limits are enriched limits for a suitable weight (or index) [St2].

A second motivation comes from the fact that bicategories are "monoidal categories with several objects". Included in this is the study of categories enriched in a bicategory which leads to a unification of category theory, sheaf theory, boolean-valued logic, and metric space theory [W, St5, St8, BCSW, P]. The generalization of **Cauchy completion** from the metric space case is fundamental [L2].

A third impetus is the formalisation of properties of the bicategory $\mathcal{C}at$ (as in the part of category theory which abstracts properties of the category $\mathcal{S}et$ of sets) allowing the use of bicategories as organisational tools for studying categories with extra structure (in the way that categories themselves organise sets with structure). This leads to the study of **arrow categories** [Gr1], **adjunctions** [K1], **monads** (= **triples**) [St0], **Kan extensions** [SW, St1], **factorization systems** [St5, St6, St7, CJSV], and the like, as concepts belonging within a fixed bicategory. Familiar constructions (such as **comma categories** and **Eilenberg-Moore categories for monads**) made with these concepts turn out to be limits of the kind arising in other motivations. In this spirit, one can mimic the construction of $\mathcal{M}od$ from $\mathcal{C}at$ starting with a bicategory (satisfying certain exactness conditions) much as one constructs a category of relations in a **regular category** or **topos** [St3, CJSV, RW]. The size needs of category theory add extra challenges to the subject [St1, SW].

Low-dimensional topology enters bicategories from two dual directions. The commutative diagrams familiar in a category laxify in a bicategory to 2-dimensional diagrams with 2-arrows in the regions; and these diagrams, if well formed, can be evaluated, using the compositions, to yield a unique 2-arrow called the *pasted composite* of the diagram [Bn1, Gr2, KS] . Two-dimensional graph-like structures called *computads* were designed to formalise pasting [St2]. The planar Poincaré-dual view replaces pasting diagrams with **string diagrams**; the 2-arrows label nodes, the arrows label strings (intervals embedded in the Euclidean plane), and the objects label regions [JS, St10]. The planar geometry of string diagrams under deformation is faithful to the algebra of bicategories. Also see **monoidal bicategories**.

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Higher-dimensional categories; n-categories

For any natural number n, an *n-category* A [Ehr] consists of sets $A_0, A_1, A_2, \ldots, A_n$, where the elements of A_m are called *m-arrows;* together with, for all $0 \le k < m \le n$, a category structure for which A_k is the set of objects and A_m is the set of arrows where the composition is denoted by $a \circ_k b$ (for composable $a, b \in A_m$); such that, for all $0 \le h < k < m \le n$, there is a **2-category** with A_h, A_k, A_m as set of objects, arrows and 2-arrows, respectively, with vertical composition $a \circ_k b$, and with horizontal composition $a \circ_h b$. The sets A_m with the source and target functions $A_m \dashrightarrow A_{m-1}$ form the underlying **globular set** (or **n-graph**) of A. For $0 \le k \le n$ and for $a, b \in A_k$ with the same (k–1)-source and (k–1)-target, there is an (n–k–1)-category A(a,b) whose m-arrows ($k < m \le n$) are the m-arrows c : a --> b of A. In particular, for 0-arrows a, b (also called *objects*), there is an (n–1)-category A(a,b) which provides the basis of an alternative definition [EK] of n-category using recursion and **enriched categories** [Kel]. It follows that there is an (n+1)-category *n-Cat* whose objects are n-categories and whose 1-arrows are *n-functors.* For infinite n, the notion of *ω-category* [Rob] is obtained. An *n-groupoid* is an n-category such that, for all $0 < m \le n$, each m-arrow is invertible with respect to the (m–1)-composition (for n infinite, *∞–groupoid* is used in [BH] rather than *ω-groupoid* by which they mean something else).

One reason for studying n-categories was to use them as coefficient objects for **non-abelian cohomology**. This required constructing the **nerve** of an n-category which, in turn, required extending the notion of **computad** to *n-computad,* defining *free n-categories* on n-computads, and formalising n-pasting [St1; Jo1; St2; Jo2; Pw1].

Ever since the appearance of **bicategories** (= **weak 2-categories**) in 1967, the prospect of **weak n-categories** (n > 2) has been contemplated with some trepidation [ML; p. 126]. The need for monoidal bicategories arose in various contexts especially in the theory of categories enriched in a bicategory [W] where it was realised that monoidal structure on the base was needed to extend results of usual enriched category theory [Kel]. The general definition of monoidal bicategory (as the one object case of *tricategory*) was not published until [GPS], however, in 1985, the structure of **braiding** [JS2] was defined on a monoidal (= tensor) category $\mathcal{V}$ and was shown to be exactly what arose when a tensor product (independent of specific axioms) was present on the one-object bicategory $\Sigma\mathcal{V}$. The connection between braidings and the **Yang-Baxter equation** was soon understood [T; JS1]. This was followed by a connection between the **Zamolodchikov equation** and braided monoidal bicategories [KV3; KV4] using more explicit descriptions of this last structure. The categorical formulation of **tangles** in terms of braiding plus **adjunction** (or duality) was then developed [FY; Sh; RT]. See [Kas] for the role this subject plays in the theory of **quantum groups**.

Not every tricategory is equivalent (in the appropriate sense) to a 3-category: the **interchange law** between 0- and 1-compositions needs to be weakened from an equality to an invertible coherent 3-cell; the groupoid case of this had arisen in unpublished work of A. Joyal and M. Tierney on algebraic homotopy 3-types in the early 1980s; details, together with the connection with **loop spaces**, can be found in [B; BFSV]. (A different non-globular higher-groupoidal homotopy n-type for all n was established in [Lo].) Whereas 3-categories are categories enriched in the category *2-Cat* of 2-categories with cartesian product as tensor product, *Gray categories* (or "semi-strict 3-categories") are categories enriched in the monoidal category *2-Cat* where the tensor product is a pseudo-version of that defined in [Gy]. The **coherence theorem** of [GPS] states that every tricategory is (tri)equivalent to a Gray category. A basic example of a tricategory is $\mathcal{B}icat$ whose objects are bicategories, arrows are **pseudo-functors**, 2-arrows are *pseudo-natural transformations,* and 3-arrows are *modifications.*

While a simplicial approach to defining weak n-categories for all n was suggested in [St1], the first precise definition was that of [BD2] announced in November 1995. Other apparently quite different definitions [Ba1; Ta] were announced in 1996 and [Joy] in 1997. Both the Baez-Dolan and Batanin definitions involve differently generalised **operads** of P. May [May] as somewhat foreshadowed by T. Trimble whose operad approach to weak n-categories had led to a definition of weak 4-category (*tetracategory*) [Tr].

With precise definitions available, the question of their equivalence is paramount. A modified version [HMP] of the Baez-Dolan definition together with generalised computad techniques from [Ba2] are expected to show the equivalence of the Baez-Dolan and Batanin definitions.

The next problem is to find the correct **coherence theorem** for weak n-categories: what are the appropriately stricter structures generalising Gray categories for n = 3? Strong candidates seem to be the *teisi* (Welsh for "stacks") of [C1; C2; C3]. Another problem is to find a precise definition of the weak (n+1)-category of weak n-categories.

The geometry of weak n-categories (n > 2) is only at its early stages [MT; F; KT; BL], however, there are strong suggestions that this will lead to constructions of invariants for higher-dimensional manifolds and have application to conformal field theory [Car; BD1; CY; Mck].

The theory of weak n-categories, even for n = 3, is also in its infancy [DS; Mar]. Reasons for developing this theory, from the computer science viewpoint, are described in [Pw2]. There are applications to **concurrent programming** and **term rewriting systems**; see [St3; St4] for references.

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