1 Questions for you before the SGTA

This section contains the problems you should attempt at home in preparation for your SGTA.

1. The sum of three consecutive numbers is 114. Find the numbers.

2. Divide $91 among three people so the first person gets $6 more than the second person, and the second person gets half as much as the third person.

3. I took my entourage to IKEA. I bought 4 chairs, a table, and 3 bookcases for $121. My lawyer bought 6 chairs, 2 tables and 2 bookcases for $148. My bodyguard bought 2 chairs, 3 tables, and a bookcase for $109. My agent bought 5 chairs, 4 tables, and 4 bookcases—how much did she spend?

4. Solve each system of equations, in 2 variables.
   Describe geometrically both the system, its solution(s) if any, and your method of solution.

   (a) $\begin{cases} 3x + y = 14 \\ 5x + 2y = 19 \end{cases}$
   (b) $\begin{cases} 3x + 5y = 11 \\ 9x + 8y = 40 \end{cases}$
   (c) $\begin{cases} 4x + 6y = 10 \\ 2x + 3y = 11 \end{cases}$

5. Find an equation for the line tangent to the graph of $y = 2x^2 - 3x + 4$ at the point on the graph where $x = 2$.

6. Find an equation for the line normal to the graph of $y = 2x^2 - 3x + 4$ at the point on the graph where $x = 2$. Note that the normal is perpendicular to the tangent.

7. Suppose that for a function $g(x)$ you know that $g(5) = 2$ and further, that $g'(x) = \dfrac{6}{1+x}$.
   Let $f(x) = x^2 g(x)$. In this question, there is no need to find a formula for $g(x)$ itself.
   Calculate $f'(5)$.

8. Find the derivative for each function. For part (a), let $s = 9x^4 + 8$ and use the formula: $\dfrac{dy}{dx} = \dfrac{dy}{ds} \times \dfrac{ds}{dx}$.

   This is the chain rule. Use a similar idea for part (b) and part (c). The other rules being used in this question are the product rule, and the quotient rule. (See section 6.2 of *Elements of Mathematics for Economics and Finance*, Mavron.)

   (a) $y = \sqrt[3]{9x^4 + 8}$
   (b) $y = x^2(6x + 7)^{17}$
   (c) $y = \dfrac{(2x+3)^{17}}{4x+5}$

2 Questions that you will work through during the SGTA

This section contains problems that you will work through during the SGTA.

1. Solve each system of equations in 3 variables.

   (a) $\begin{cases} x - 2y + 3z = 1 \\ x + 2y - z = 13 \\ 3x + 2y - 5z = 3 \end{cases}$
   (b) $\begin{cases} x - y + 5z = -2 \\ 2x + y + 4z = 2 \\ 2x + 4y - 2z = 8 \end{cases}$
   (c) $\begin{cases} 4x + 13y + 6z = 14, \\ 2x + 7y + 4z = 7 \\ 3x + 10y + 5z = 11 \end{cases}$

2. For Q8(c) above (the derivative of a quotient), find the x values for which $y' = 0$. There are two of them. If you were to zoom in on the graph at either of these points, what should the graph look like?

3. Find equations for the lines through $(1, 1)$ and tangent to the graph of $y = 2x^2 + x + 6$.

4. Find equations for the lines through $(-8, 4)$ and normal to the graph of $y = x^2 + x - 4$.

5. For cost $C(q)$ and revenue $R(q)$ functions for an item, if $C'(500) = 75$ and $R'(500) = 100$, we ask should the quantity produced be increased or decreased from $q = 500$ in order to increase profits?

3 Additional problems

These are problems that students who would like something a little more challenging can try at home after the SGTA. Your instructor may discuss some of these problems in the SGTA if time permits.

1. A single linear equation in three variables is represented geometrically as a plane in three dimensions. The solution of three linear equations in three variables is then the common intersection of three planes. Draw diagrams representing the several ways this can occur. One example is the three planes being the same plane, so the answer would be a plane of solutions.

2. Market research suggests that if tickets to a series of concerts are priced at \$150 per ticket, the total number of tickets that can be sold will be 30 000. For each \$1 rise in the price of the tickets, the market researchers predict that 150 fewer tickets will be sold.

    (a) If the price that is set for the tickets is \$$p$, write down an equation for the number of tickets $N(p)$ that will be sold.

    (b) Hence, find an equation for $R(p)$, the total revenue from ticket sales if the price is p per ticket.

    (c) The ticket price that produces the most revenue from the concert series is the price p such that $R'(p) = 0$. Find this value of p. How much revenue will be generated with this ticket price?

4   Home-work problems

These are the questions that you need to do and hand to your tutor at the following week's tutorial.
Use a barcoded sheet, obtained from a link on iLearn in the 'Tutorials' section. Submissions without a coversheet
*will **not** be marked.*

Marking guidelines   8 marks total.
There are 6 marks for correctness of the logic and mathematical content. Submissions that could be considered
a first draft, that is too messy or unorganized, will **not** be marked.
The other 2 marks are for presentation, as follows.

2 marks.   Clear and well explained.

1 mark.   Not completely clear, or limited, reasons.

0 marks.   Little or no attempt at an explanation; e.g. calculations only or only answers.

Do not write your answers on this page. Write your answers on a separate sheet of paper, and attach the relevant barcoded cover sheet.

1. (1 mark) One solution of $3x^2 + kx - 7 = 0$ is $x = 4$. Find the other solution.

2. (2 marks) Solve the following linear system, and describe the result geometrically. $\begin{cases} 3x + 5y = 19 \\ 5x - 2y = 11 \end{cases}$

3. (3 marks) Find the equation of the tangent to $f(x) = \dfrac{x^3}{2} - \dfrac{4}{3x}$ at $x = 2$. Use technology to zoom in on the graph at $x = 2$ to verify your result. Various options for technology include gnuplot, wolframalpha, desmos, and geogebra. Here's a link for wolfram.