Theory and Application of Stochastic Unification-based Grammars

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Talk outline

- Motivation for and applications of stochastic grammars
- Discriminative training of stochastic grammars
  - supervised training from parsed corpora
  - unsupervised training from sentence-aligned bitext
- Avoiding enumerating parses
  - Packed parse representations
  - Feature locality
  - Dynamic programming using graphical model techniques

Why combine grammars and statistics?

- Language is used to *convey information*
  - Grammars capture the *form-meaning mapping*
- Interpretation is dependent on *many interacting factors*
  - Grammar is about expressing linguistic constraints
- *Ambiguity* is pervasive in language
  - Statistics is the theory of *inference under uncertainty*
- Learning the grammar of a language is a prerequisite
  - Language learning is a statistical inference problem

What can we do with SUBGs?

- Identify most likely parses (focused information retrieval)
- Machine translation (find most likely translation)
- Language modelling for speech recognition and OCR
  - Requires joint models

Two problems of non-statistical CL

1. *Ambiguity explodes combinatorially*

   (162) *Even though it's possible to scan using the Auto Image Enhance mode, it's best to use the normal scan mode to scan your documents.*

   - Refining the grammar is often self-defeating
     $\Rightarrow$ splits states $\Rightarrow$ makes the problem worse!
   - Preference information guides parser to correct analysis

2. *Requiring linguistic well-formedness leads to non-robustness*

   - Perfectly comprehensible sentences receive no parses

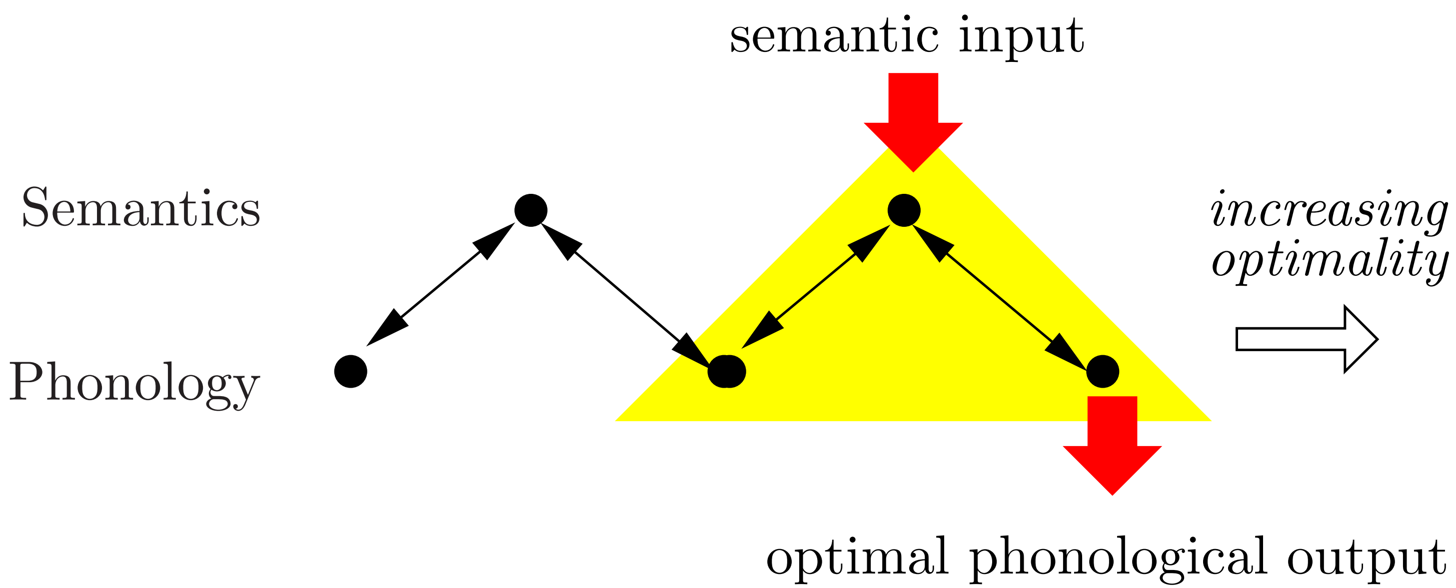
Conventional approaches to robustness

- We want to be able to analyse ill-formed input, e.g. *He walk.*
  - Ignoring agreement $\Rightarrow$ spurious ambiguity
    I saw the father of the children that speak(s) French
- Extra-grammatical rules, repair mechanisms, . . .
  - How can semantic interpretation take place without a well-formed syntactic analysis?
- A preference-based approach provides a systematic treatment of robustness too!

Generation with ranked analyses

- Probability distribution over phonology/semantics pairs $\omega \in \Omega$
- Generation optimizes conditional probability of phonological output *given the semantic input* s.

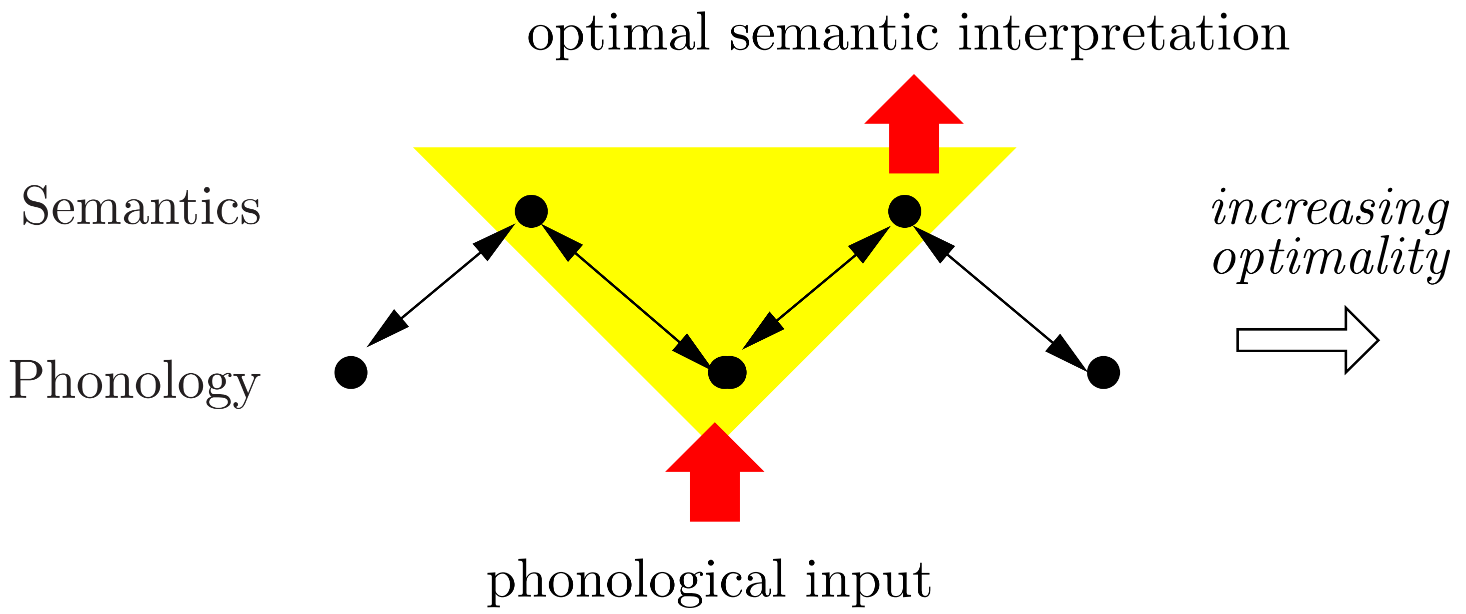
$$\text{Generate}(s) \;=\; \operatorname*{argmax}_{\omega} \mathrm{P}(\omega \mid \text{semantics}(\omega) = s)$$



Parsing with ranked analyses

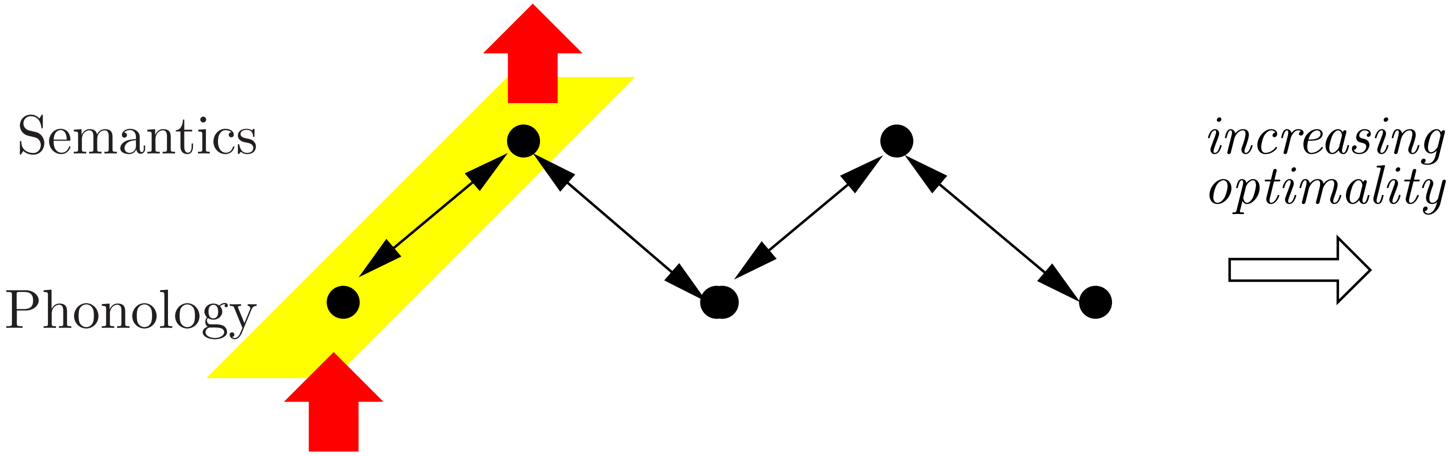
- Parsing optimizes the conditional probability of the semantics *given the phonological form* p

$$\mathrm{Parse}(p) \;=\; \operatorname*{argmax}_{\omega} \mathrm{P}(\omega \mid \mathrm{phonology}(\omega) = p)$$



Robustness and ranked interpretations

- Parsing and generation involve *different* conditional distributions!
- Grammar pairs "ungrammatical" sentences with interpretations



never generated, but interpretable!

Learning & comprehension involves inference

- Both language learning and language comprehension require identifying "hidden" properties of the input
- The input is (apparently) compatible with different hidden structures
- Statistical inference may suceed even if there is insufficient information for deductive approaches
- Ranked analyses provide a systematic treatment of preferences and robustness

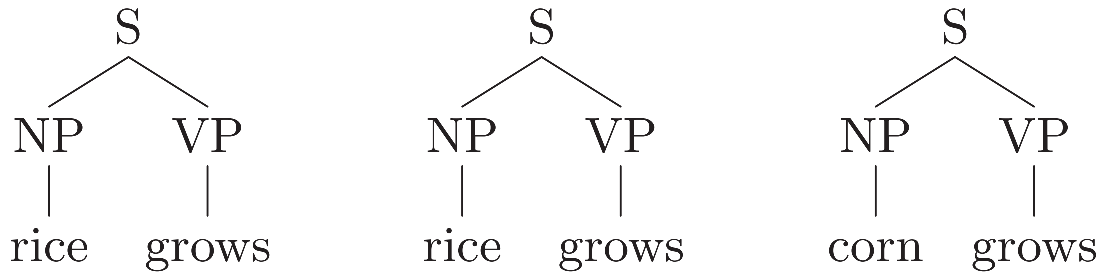
Linguistic knowledge and statistical parsing

- Statistical parsers are *not* “linguistics-free”
  - Conditioning features
  - Syntactic annotations in training data
- *What is the most effective way to import useful linguistic knowledge?*
  - *manually* specify possible linguistic structures
  - manually specify statistical features
  - learn feature weights from training data

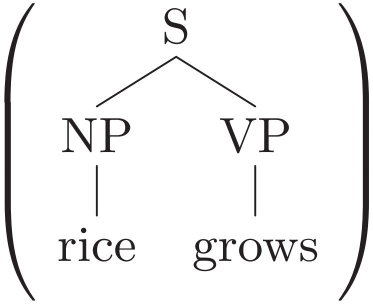
Statistical learning and parsing

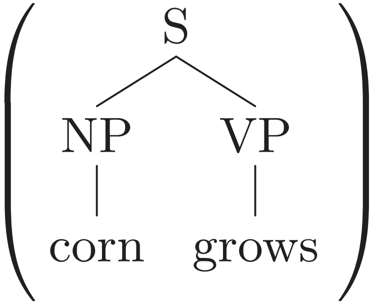
- Grammar defines (universally) possible linguistic structures Ω
- Family of probability distributions P_θ on Ω parameterized by θ
- Learning involves finding θ which makes the input most likely
- Given θ and a yield (terminal string) y, parsing involves finding most probable structure in $\{\omega | Y(\omega) = y\}$
- How can we define such probability distributions?
- Computationally efficient inference?

PCFGs and relative frequency estimator

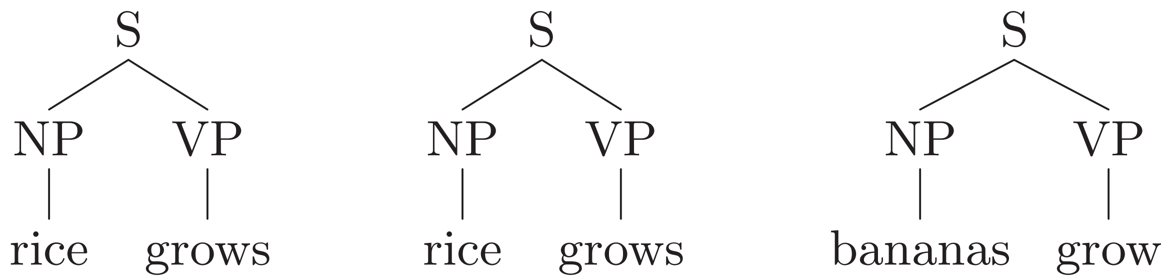


rule	count	rel freq
S $\rightarrow$ NP VP	3	1
NP $\rightarrow$ rice	2	2/3
NP $\rightarrow$ corn	1	1/3
VP $\rightarrow$ grows	3	1

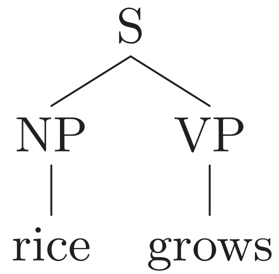
P  = 2/3

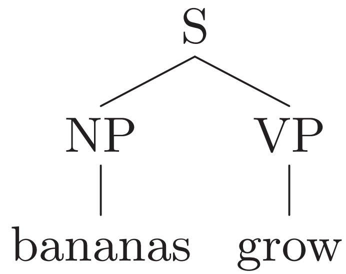
P  = 1/3

Non-local constraints



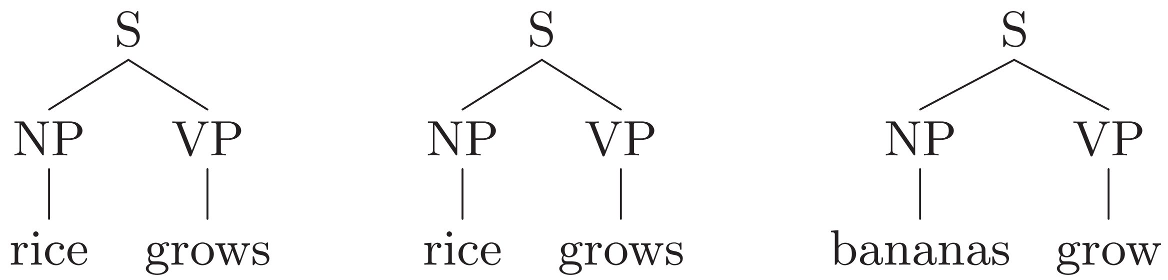
rule	count	rel freq
S → NP VP	3	1
NP → rice	2	2/3
NP → bananas	1	1/3
VP → grows	2	2/3
VP → grow	1	1/3

P () = 4/9

P () = 1/9

Z = 5/9

Renormalization



rule	**count**	**rel freq**
S → NP VP	3	1
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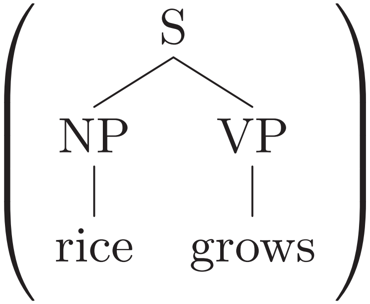
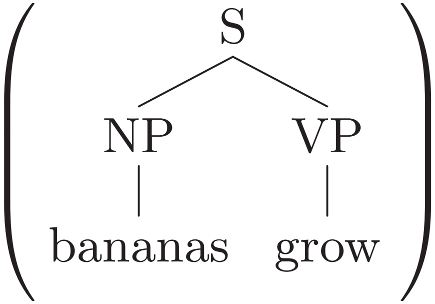
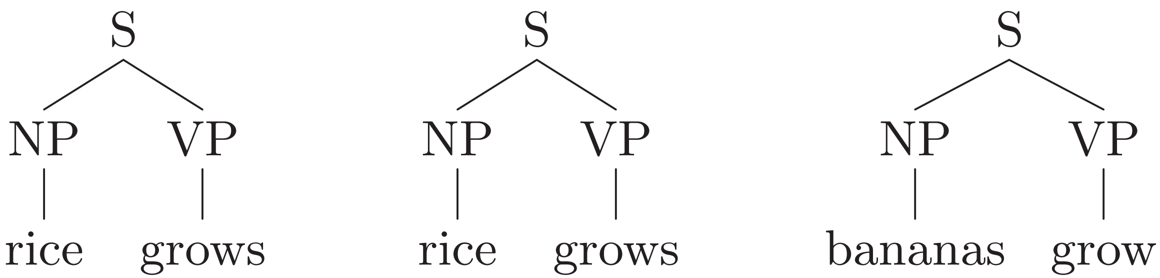
NP → bananas	1	1/3
VP → grows	2	2/3
VP → grow	1	1/3

P  = 4/9 4/5

P  = 1/9 1/5

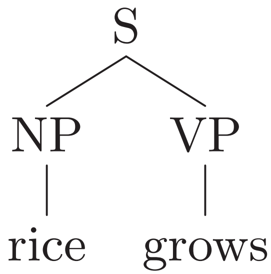
$Z = 5/9$

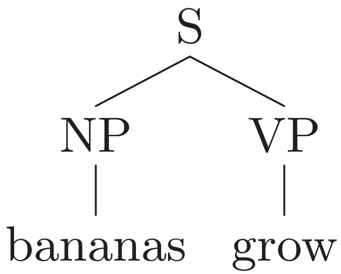
Other values do better!



rule	**count**	**rel freq**
S → NP VP	3	1
NP → rice	2	2/3
NP → bananas	1	1/3
VP → grows	2	1/2
VP → grow	1	1/2

(Abney 1997)

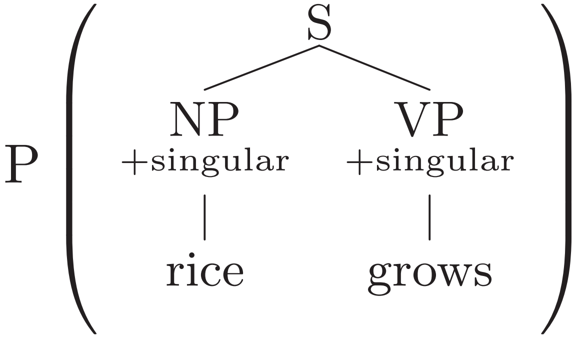
P() = 2/6 2/3

P() = 1/6 1/3

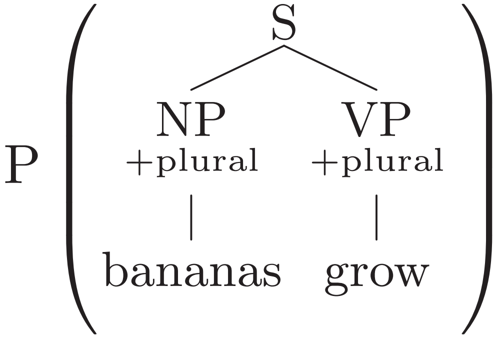
Z = 3/6

Make dependencies local – GPSG-style

rule	**count**	**rel freq**
$S \rightarrow \underset{+\text{singular}}{NP} \; \underset{+\text{singular}}{VP}$	2	2/3
$S \rightarrow \underset{+\text{plural}}{NP} \; \underset{+\text{plural}}{VP}$	1	1/3
$\underset{+\text{singular}}{NP} \rightarrow \text{rice}$	2	1
$\underset{+\text{plural}}{NP} \rightarrow \text{bananas}$	1	1
$\underset{+\text{singular}}{VP} \rightarrow \text{grows}$	2	1
$\underset{+\text{plural}}{VP} \rightarrow \text{grow}$	1	1



$= 2/3$



$= 1/3$

Summary

All dependencies are local or context-free:

- rules are "natural" features of probability distribution
- relative rule frequency is MLE

Structures exhibit non-local dependencies:

- no easy way to obtain "natural" features
- with renormalization, relative frequency estimator is not MLE
  - MLE is much more complicated
- this estimator handles non-rule and rule features

  ⇒ *no need to restrict attention to rule features*

Log linear models

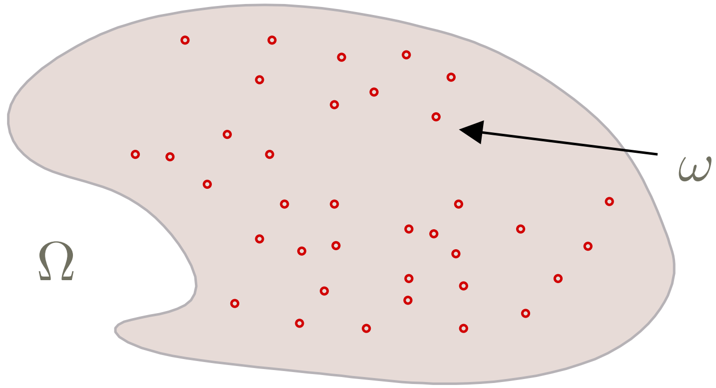
- The *log likelihood* is a *linear* function of feature values
- Ω = set of syntactic structures (not necessarily trees)
- $f_j(\omega)$ = number of occurences of jth feature in $\omega \in \Omega$ (feature $\neq$ attribute)
- λ_j are "feature weight" parameters

$$W_\lambda(\omega) = \exp(\sum_{j=1}^{m} \lambda_j f_j(\omega))$$

$$Z_\lambda = \sum_{\omega \in \Omega} W_\lambda(\omega)$$

$$\mathrm{P}_\lambda(\omega) = \frac{W_\lambda(\omega)}{Z_\lambda}$$

$$\log \mathrm{P}_\lambda(\omega) = \sum_{j=1}^{m} \lambda_j f_j(\omega) - \log Z_\lambda$$

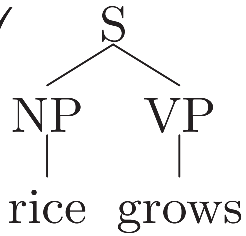


PCFGs are log-linear models

Ω = set of all trees generated by G

$f_j(\omega)$ = number of times the jth rule is used in $\omega \in \Omega$

θ_j = probability of jth rule in G $\qquad \lambda_j = \log \theta_j$



$$f\left(\text{tree}\right) = [\underbrace{1}_{\text{S}\to\text{NP VP}}, \underbrace{1}_{\text{NP}\to\text{rice}}, \underbrace{0}_{\text{NP}\to\text{bananas}}, \underbrace{1}_{\text{VP}\to\text{grows}}, \underbrace{0}_{\text{VP}\to\text{grow}}]$$

$$\mathrm{P}_\theta(\omega) = \prod_{j=1}^{m} \theta_j^{f_j(\omega)} = \exp(\sum_{j=1}^{m} \lambda_j f_j(\omega)) \quad \text{where } \lambda_j = \log \theta_j$$

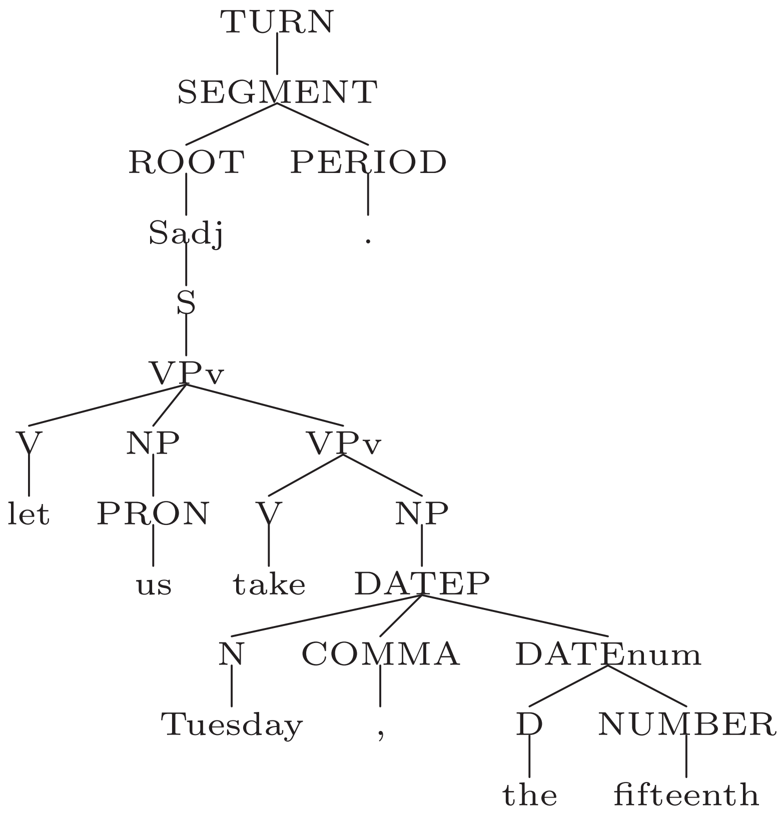
Stochastic Lexical-Functional Grammar

- Unification-based grammar (competence) defines well-formed syntactic structures Ω
  - In SLFG, these are c-structure/f-structure pairs
- Stochastic model (performance) defines a probability distribution over Ω
  - Features $f_1, \ldots, f_m$, where each f_j maps each $\omega \in \Omega$ to a feature occurence count $f_j(\omega)$
  - Probability distribution defined by log linear model

$$\log \mathrm{P}_\lambda(\omega) = \sum_{j=1}^{m} \lambda_j f_j(\omega) - \log Z_\lambda$$

- Same approach applies to virtually any theory of grammar

Sample parses




```
SENTENCE_ID  BAC002_E
  OBJ         [ ANIM +
                CASE ACC
                NUM PL
                PERS 1
                PRED PRO
                PRON-FORM WE
                PRON-TYPE PERS ] 9
  PASSIVE –
  PRED  LET⟨2,10⟩9
  STMT-TYPE IMPERATIVE
  SUBJ        [ PERS 2
                PRED PRO
                PRON-TYPE NULL ] 2
  TNS-ASP [ MOOD IMPERATIVE ]
  XCOMP   OBJ [ ANIM –
                APP [ NTYPE [ NUMBER ORD
                              TIME DATE ]
                      NUM SG
                      PRED fifteen
                      SPEC [ SPEC-FORM THE
                             SPEC-TYPE DEF ] ]
                CASE ACC
                GEND NEUT
                NTYPE [ GRAIN COUNT
                        PROPER DATE
                        TIME DAY ]
                NUM SG
                PERS 3
                PRED  TUESDAY ] 13
          PASSIVE –
  22
```

Features used

Rule features: For every non-terminal X, $f_X(\omega)$ is the number of times X occurs in c-structure of ω

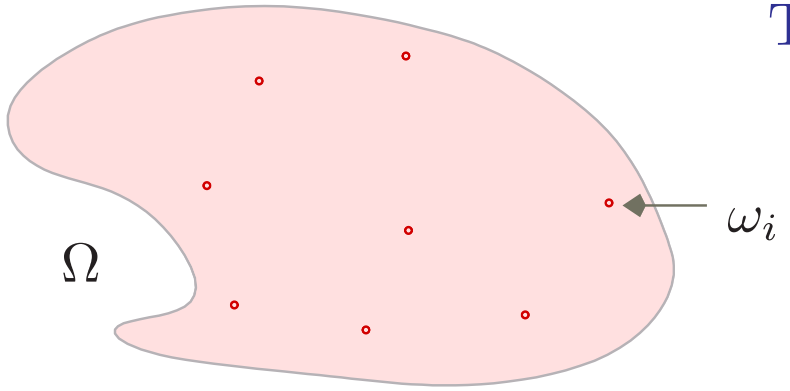
Attribute value features: For every attribute a and every atomic value v, $f_{a=v}(\omega)$ is the number of times the pair $a = v$ appears in ω

Argument and adjunct features: For every grammatical function g, $f_g(\omega)$ is the number of times that g appears in ω

Other features: Dates, times, locations; right branching; attachment location; parallelism in coordination; . . .

Features are *not* independent, but dependency structure is unknown.

ML estimation for log linear models



Training data $D = \omega_1, \ldots, \omega_n$

$$\hat{\lambda} = \operatorname*{argmax}_{\lambda} L_D(\lambda)$$

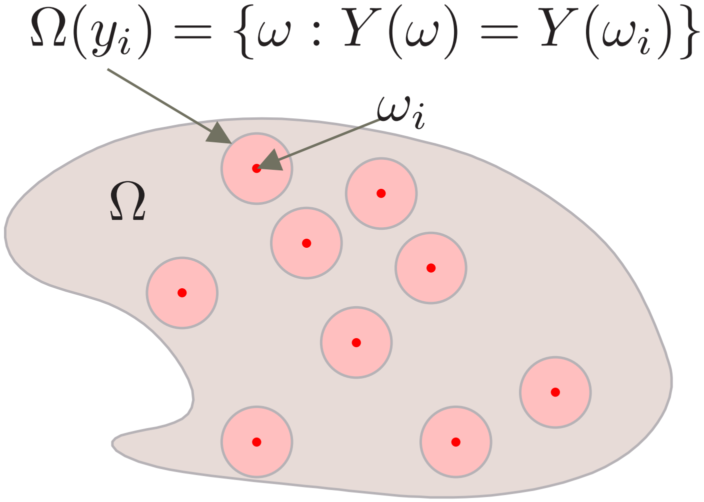
$$L_D(\lambda) = \prod_{i=1}^{n} \mathrm{P}_\lambda(\omega_i)$$

$$\mathrm{P}_\lambda(\omega) = \frac{W_\lambda(\omega)}{Z_\lambda} \quad W_\lambda(\omega) = \exp(\sum_j \lambda_j f_j(\omega)) \quad Z_\lambda = \sum_{\omega' \in \Omega} W_\lambda(\omega')$$

- For a PCFG, $\hat{\lambda}$ is easy to calculate, but …
- in general $\partial L_D / \partial \lambda_j$ and Z_λ are *intractable analytically and numerically*
- Abney (1997) suggests a Monte-Carlo calculation method

Pseudo-likelihood

The *pseudo-likelihood* of ω is the *conditional probability* of the *hidden part* (syntactic structure) ω given its *visible part* (yield or terminal string) $y = Y(\omega)$ (Besag 1974)



$$\begin{aligned}
\hat{\lambda} &= \operatorname*{argmax}_{\lambda} \mathrm{PL}_D(\lambda) \\
\mathrm{PL}_D(\lambda) &= \prod_{i=1}^{n} \mathrm{P}_\lambda(\omega_i | y_i) \\
\mathrm{P}_\lambda(\omega | y) &= \frac{W_\lambda(\omega)}{Z_\lambda(y)}
\end{aligned}$$

$$W_\lambda(\omega) = \exp(\sum_j \lambda_j f_j(\omega)) \qquad Z_\lambda(y) = \sum_{\omega' \in \Omega(y)} W_\lambda(\omega')$$

Pseudo-likelihood versus likelihood

- The pseudo-partition function $Z_\lambda(y)$ is *much easier to compute* than the partition function Z_λ
  - Z_λ requires a sum over Ω
  - $Z_\lambda(y)$ requires a sum over Ω_y (parses of y)
- Maximum *likelihood* estimates full joint distribution
  - learns distribution of both yields and parses given yields
- Maximum *pseudo-likelihood* estimates a conditional distribution
  - learns distribution of *parses given yields*, but not yields
  - conditional distribution is what you need for parsing
  - cognitively more plausible?
- Maximizing pseudo-likelihood *does not* maximize likelihood
  - PL estimator is *consistent for the conditional distribution*

Pseudo-likelihood estimation

	Correct parse's features	**All other parses' features**
sentence 1	$[1, 3, 2]$	$[2, 2, 3]$ $[3, 1, 5]$ $[2, 6, 3]$
sentence 2	$[7, 2, 1]$	$[2, 5, 5]$
sentence 3	$[2, 4, 2]$	$[1, 1, 7]$ $[7, 2, 1]$
...	...	...

- Training data is *fully observed* (i.e., parsed data)
- Choose λ to maximize (log) likelihood of *correct* parses relative to other parses
- Distribution of *sentences* is ignored

Pseudo-constant features are uninformative

	Correct parse's features	**All other parses' features**
sentence 1	$[1, 3, \mathit{2}]$	$[2, 2, \mathit{2}]$ $[3, 1, \mathit{2}]$ $[2, 6, \mathit{2}]$
sentence 2	$[7, 2, \mathit{5}]$	$[2, 5, \mathit{5}]$
sentence 3	$[2, 4, \mathit{4}]$	$[1, 1, \mathit{4}]$ $[7, 2, \mathit{4}]$
...	...	...

- *Pseudo-constant features* are identical within every set of parses
- They contribute the same constant factor to each parses' likelihood
- They do not distinguish parses of any sentence $\Rightarrow$ irrelevant

Pseudo-maximal features ⇒ unbounded $\widehat{\lambda_j}$

	Correct parse's features	**All other parses' features**
sentence 1	[1, *3*, 2]	[2, *3*, 4] [3, *1*, 1] [2, *1*, 1]
sentence 2	[2, *7*, 4]	[3, *7*, 2]
sentence 3	[2, *4*, 4]	[1, *1*, 1] [1, *2*, 4]

- A *pseudo-maximal feature* always reaches its maximum value within a parse on the correct parse
- If f_j is pseudo-maximal, $\widehat{\lambda_j} \to \infty$ (hard constraint)
- If f_j is pseudo-minimal, $\widehat{\lambda_j} \to -\infty$ (hard constraint)

Regularization

- f_j is pseudo-maximal over training data $\not\Rightarrow$ f_j is pseudo-maximal over all of Ω (sparse data)

- Regularization: add *bias* term to ensure optimal λ_j is finite
  Multiply the pseudo-likelihood by a zero-mean normal with diagonal covariance

$$\hat{\lambda} = \operatorname*{argmax}_{\lambda} \log \mathrm{PL}_D(\lambda) - \sum_{j=1}^{m} \frac{\lambda_j^2}{2\sigma_j^2}$$

  where σ_j is 7 times the maximum value of f_j found in the corpus

Stochastic LFG experiment

- Two parsed LFG corpora provided by Xerox Parc
- Grammars unavailable, but corpus contains all parses and hand-identified correct parse
- Features chosen by inspecting Verbmobil corpus only

	Verbmobil corpus	Homecentre corpus
# of sentences	540	980
# of ambiguous sentences	324	424
Av. length of ambig. sentences	13.8	13.1
# of parses	3245	2865
# of features	191	227
# of rule features	59	57

Pseudo-likelihood estimator evaluation

	Verbmobil corpus 324 sentences		**Homecentre corpus** 424 sentences	
	C	$-\log \mathrm{PL}$	C	$-\log \mathrm{PL}$
Baseline estimator	88.8	533.2	136.9	590.7
Pseudo-likelihood estimator	180.0	401.3	283.25	580.6

- Test corpus only contains sentences with more than one parse
- C is the number of maximum likelihood parses of held-out test corpus that were the correct parses
- 10-fold cross-validation evaluation
- Combined system performance: 75% of MAP parses are correct

What have we achieved?

+ Log linear framework applies to *any* theory of grammar
+ Pseudo-likelihood estimator is practical for grammars with thousands of analyses/sentence
+ Features can be anything "read off" a structure
+ Systematic treatment of preferences

? Where's the linguistic structure gone? Probability distribution determined solely by feature count vector

– Parser is just as non-robust or "brittle" as before

$\Rightarrow$ Re-express hard linguistic constraints as soft constraints

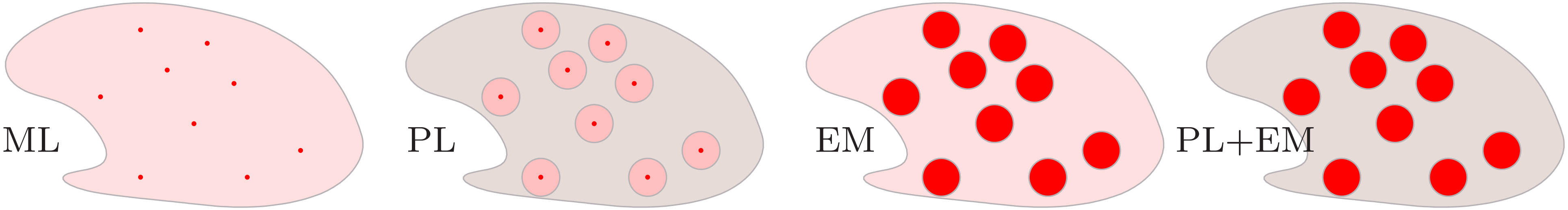
Summary

- Log-linear models provide a general way of defining probability distributions in the face of context-sensitive dependencies
- The pseudo-likelihood estimator is computationally tractable for realistic LFGs
- Auxiliary distributions provide a principled way of incorporating other distributional information
- The combined LFG parser + log linear model obtains the correct parse on 73% of Verbmobil and almost 80% of Homecentre corpus sentences

PL estimation and hidden data

- PL estimation *ignores* distribution of strings

$\Rightarrow$ Cannot learn from strings alone



	maximizes likelihood of	**relative to**
ML	ω_i	Ω
PL	ω_i	$\Omega(y_i)$
EM	$\Omega(y_i)$	Ω
PL+EM	$\Omega(y_i)$	$\Omega(y_i)$

Psychologically-realistic conditional models

- *Joint* models $\mathrm{P}(\omega)$ predict *what is said* and *how it is said*
- *Modularity:* These two processes are very different!
- Conditional models in SUBGs: $\mathrm{P}(S|Y)$
  ($S =$ semantics, $Y =$ phonology)
- A *psycholinguistically realistic* statistical model
  – World model: $\mathrm{P}(S)$
  – Linguistic model: $\mathrm{P}(Y|S)$
- Parsing with such models:

$$\mathrm{P}(S|Y) \propto \mathrm{P}(Y|S)\mathrm{P}(S)$$

Language acquisition as parameter estimation

- Ω contains every sentence structure from every possible human language
- Each type of syntactic construction is associated with a parameter

Verb initial	$\lambda_{\mathrm{VI}} > 0$	$[_\mathrm{S}$ Kim $[_\mathrm{VP}$ will love Sandy$]]$
Verb final	$\lambda_{\mathrm{VF}} > 0$	$[_\mathrm{S}$ Kim $[_\mathrm{VP}$ Sandy love will$]]$
Verb second	$\lambda_{\mathrm{V2}} > 0$	$[_\mathrm{S}$ Kim will $[_\mathrm{VP}$ Sandy love$]]$

- Learning a language involves learning which constructions it possesses

PL estimation is cognitively unnatural

- PL estimation requires *parsed input*
  - Correct parse of “NP V NP” identifies λ_{V2} value

    $[_{\mathrm{S}}$ Kim loves $[_{\mathrm{VP}}$ Sandy$]]$ $\Rightarrow$ $\lambda_{\mathrm{V2}} > 0$

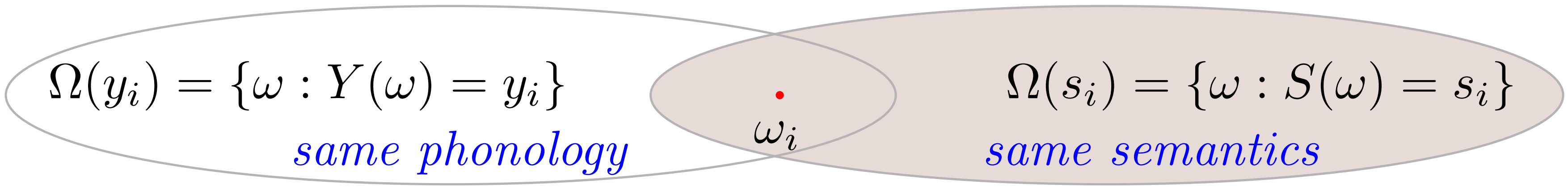
    $[_{\mathrm{S}}$ Kim $[_{\mathrm{VP}}$ loves Sandy$]]$ $\Rightarrow$ $\lambda_{\mathrm{V2}} < 0$
  - Unrealistic to assume child has access to parsed input
- PL estimator only learns from *ambiguous sentences*
  - $[_{\mathrm{S}}$ Kim $[_{\mathrm{VP}}$ Sandy love will$]]$ is uninformative to PL
- But *unambiguous sentences* are sometimes most informative!

Components of a representation

- A representation projects several components (random variables)
  - yield $Y(\omega)$, semantics $S(\omega)$
- Pseudo-likelihood can be defined with respect to each of these
  - $\Omega(y) = \{\omega | Y(\omega) = y\}$ and $\Omega(s) = \{\omega | S(\omega) = s\}$ are small and enumerable for many grammars

  $\Rightarrow$ estimation is computationally feasible
- These sets can be used to define a wide variety of estimators

Semantic pseudo-likehood

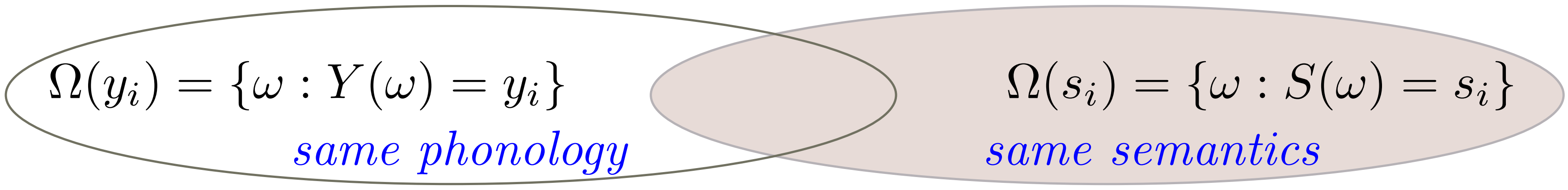


- Assume learner has access to semantics s_i and correct parse ω_i
- Treat the *semantics* s_i as visible component
- Pseudo-likelihood with *semantic comparison set*

$$\mathrm{PL}'_D(\lambda) \;=\; \prod_{i=1}^{n} \mathrm{P}_\lambda(\omega_i|s_i)$$

- Learns when a semantics can be expressed in several ways cross-linguistically
  (love(Sandy, Sasha)) $\Rightarrow^{+}$ [$_{\mathrm{S}}$ Sandy [$_{\mathrm{VP}}$ Sasha love]]) $\;\Rightarrow\;$ $\lambda_{\mathrm{VF}} > 0$

Partially observed data



- Phonology and semantics are both visible
  Training data $D' = \langle y_1, s_1 \rangle, \ldots, \langle y_n, s_n \rangle$

- Maximize the semantic pseudo-likehood of the phonology

$$\mathrm{PL}_{D'}(\lambda) \;=\; \prod_{i=1}^{n} \mathrm{P}_\lambda(y_i | s_i)$$

- Learns whenever a semantics has several yields cross-linguistically
  $(\mathrm{Fut}(\mathrm{love}(\mathrm{Sandy}, \mathrm{Sasha})) \Rightarrow^{+}$ "Sandy will Sasha love"$) \;\Rightarrow\; \lambda_{\mathrm{V2}} > 0$

Learning from aligned bilingual corpora

- Adjust models λ_a, λ_b to maximize probability that *each translation pair receives same semantic interpretation*
- Training data $D = (y_{a,1}, y_{b,1}), \ldots, (y_{a,n}, y_{b,n})$

$$(\widehat{\lambda_a}, \widehat{\lambda_b}) = \operatorname*{argmax}_{\lambda_a, \lambda_b} L_D(\lambda_a, \lambda_b)$$

$$L_D(\lambda_a, \lambda_b) = \prod_{i=1}^{n} \mathrm{P}_{\lambda_a} \times \mathrm{P}_{\lambda_b}(S_a = S_b | y_{a,i}, y_{b,i})$$

$$\mathrm{P}_{\lambda_a} \times \mathrm{P}_{\lambda_b}(s_a, s_b | y_a, y_b) = \mathrm{P}_{\lambda_a}(s_a | y_a) \mathrm{P}_{\lambda_b}(s_b | y_b)$$

- More sophisticated models are possible! (c.f., co-training)

Hidden data and bidirectional optimization

- Assume that $\mathrm{P}(S|Y)$ and $\mathrm{P}(Y|S)$ are highly skewed

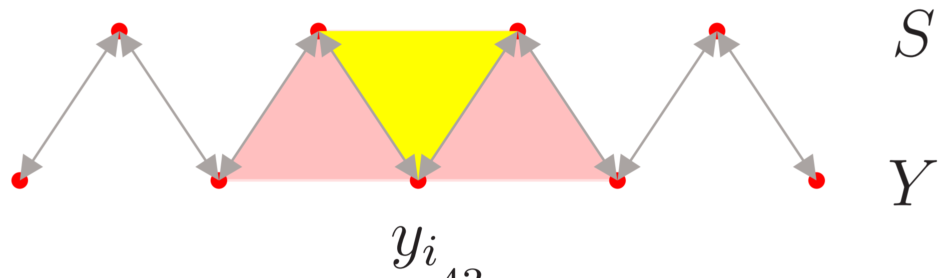
  $\Rightarrow$ Most sentences have one highly preferred interpretation

  $\Rightarrow$ Most semantics have one highly preferred sentence

- Adjust λ to maximize probability of *generating the observed string from its likely interpretations*

$$D = y_1, \ldots, y_n$$

$$PL_D(\lambda) = \prod_{i=1}^{n} \sum_{s} \mathrm{P}_\lambda(y_i|s)\mathrm{P}_\lambda(s|y_i)$$



Summary

- Log linear models provide a general framework for defining probability distributions over linguistic representations
- Joint models are difficult/impossible to estimate
- Conditional models (conditioning on the yield) are easier to estimate
- Learning conditional models from hidden data is difficult
- It may be useful to condition on the semantics
- There are many other interesting conditional models to investigate!

Parsing and estimation from packed parses

- Maxwell and Kaplan packed parse representations
- Feature locality (e.g., a f-structure constant)
- Parsing/estimation statistics are sum/max of products
- Graphical representation of product expressions
- Sum/max computations over graphs
- Other applications
  - Importance sampling
  - Best-first parsing

Reparameterization of log linear models

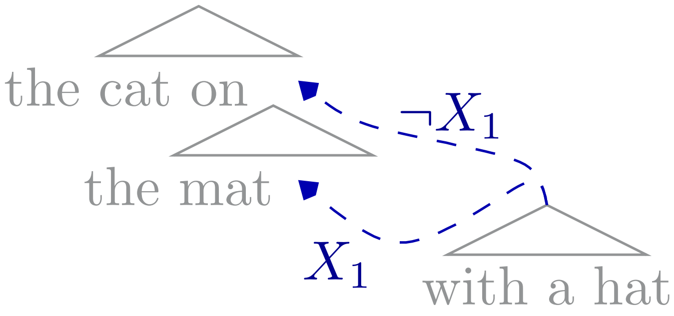
$$
\begin{aligned}
\theta_j &= \exp \lambda_j \\
W_\theta(\omega) &= \prod_{j=1}^{m} \theta_j^{f_j(\omega)} \\
\mathrm{P}_\theta(\omega|y) &= \frac{W_\theta(\omega)}{Z_\theta(y)} \\
Z_\theta(y) &= \sum_{\omega' \in \Omega(y)} W_\theta(\omega')
\end{aligned}
$$

- Change of variables permits zero probability events
- $Z_\theta(y)$ involves summing over all possible parses
- Same kind of technique finds most likely parse and calculates $E_\theta[f_j|y]$

Maxwell and Kaplan packed parses

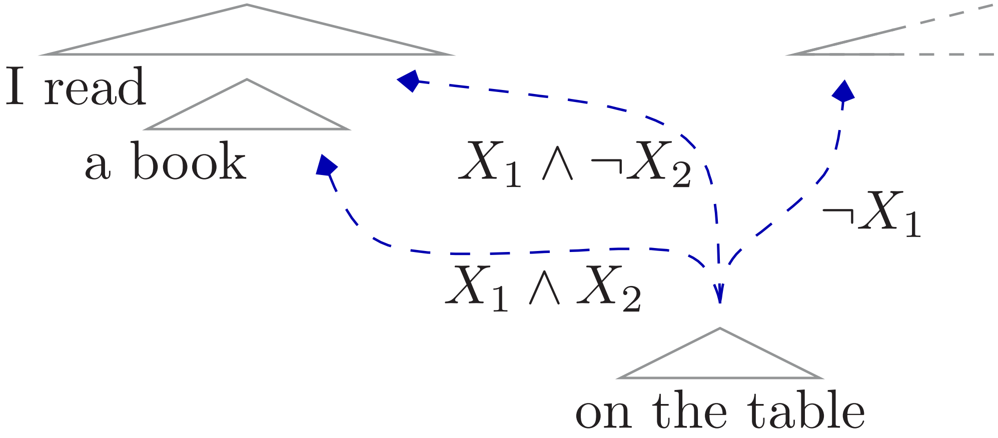
- A parse ω consists of set of fragments $\xi \in \omega$
- A fragment is in a parse when its *context function* is true
- Context functions are functions of zero or more *context variables*
- The variable assignment must satisfy "not no-good" functions
- Each parse is identified by a *unique context variable assignment*

$$\begin{aligned} y &= \text{“\textit{the cat on the mat}”} \\ y_1 &= \text{“\textit{with a hat}”} \\ X_1 &\rightarrow \text{“attach } y_1 \text{ low”} \\ \neg X_1 &\rightarrow \text{“attach } y_1 \text{ high”} \end{aligned}$$



Packed parse example

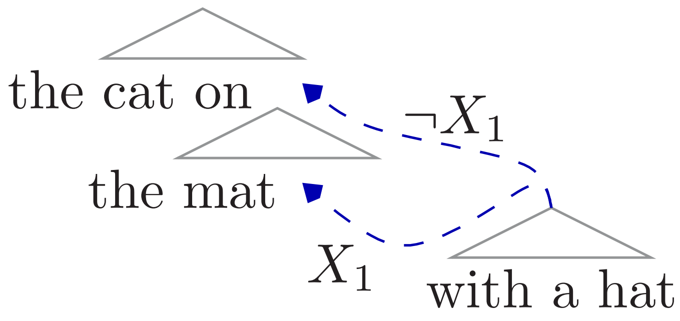
$$
\begin{aligned}
y &= \text{“}\textit{I read a book}\text{”} \\
y_1 &= \text{“}\textit{on the table}\text{”} \\
X_1 \wedge X_2 &\rightarrow \text{“attach } y_1 \text{ low”} \\
X_2 \wedge \neg X_2 &\rightarrow \text{“attach } y_1 \text{ high”} \\
\neg X_1 &\rightarrow \text{“attach } y_1 \text{ elsewhere”} \\
& X_1 \vee X_2
\end{aligned}
$$



Feature locality

- Features *local* to fragments: $f_j(\omega) = \sum_{\xi \in \omega} f_j(\xi)$

$$
\begin{aligned}
y &= \textit{“the cat on the mat”} \\
y_1 &= \textit{“with a hat”} \\
X_1 &\rightarrow \text{“attach } y_1 \text{ low”} \;\wedge\; (y_1 \;\textsc{attach}) = \textsc{low} \\
\neg X_1 &\rightarrow \text{“attach } y_1 \text{ high”} \;\wedge\; (y_1 \;\textsc{attach}) = \textsc{high}
\end{aligned}
$$



Feature locality decomposes W_θ

- Feature locality: the weight of a parse is the product of the weights of its fragments

$$W_\theta(\omega) \quad = \quad \prod_{\xi \in \omega} W_\theta(\xi)$$

$$\begin{array}{rcl}
 & & W_\theta(y \;=\; \text{``}\textit{the cat on the mat}\text{''}) \\
 & & W_\theta(y_1 \;=\; \text{``}\textit{with a hat}\text{''}) \\
X_1 & \rightarrow & W_\theta\,(\text{``attach } y_1 \text{ low''} \;\wedge\; (y_1 \;\textsc{attach}) = \textsc{low}\;) \\
\neg X_1 & \rightarrow & W_\theta\,(\text{``attach } y_1 \text{ high''} \;\wedge\; (y_1 \;\textsc{attach}) = \textsc{high}\;)
\end{array}$$

W_θ as a function of X

- Identify each parse ω by its corresponding variable assignment x
- Then $W_\theta(X) = \prod_{A \in \mathcal{A}} A(X)$,
  - Each line $\alpha(X) \to \xi$ introduces a term $W_\theta(\xi)^{\alpha(X)}$
  - A "not no-good" $\eta(X)$ introduces a term $\eta(X)$
  - Each line is a function of a subset of the variables X

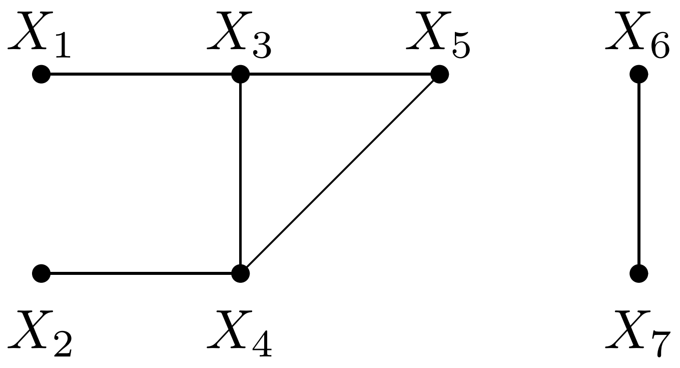
$$
\begin{array}{ccccc}
 & \vdots & & & \vdots \\
\alpha(X) & \to & \xi & \times & W_\theta(\xi)^{\alpha(X)} \\
 & \vdots & & \times & \vdots \\
 & & \eta(X) & \times & \eta(X) \\
 & \vdots & & \times & \vdots
\end{array}
$$

Dependency structure graph $\mathcal{G}_{\mathcal{A}}$

$$Z_\theta(y) \;=\; \sum_{x \in \mathcal{X}} W_\theta(x) \;=\; \sum_{x \in \mathcal{X}} \prod_{A \in \mathcal{A}} A(x)$$

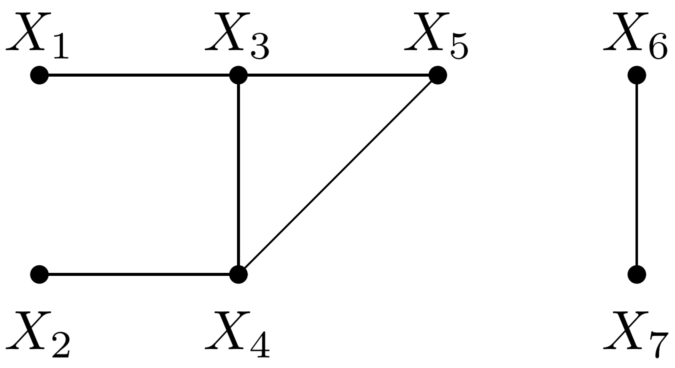
- $\mathcal{G}$ is the *dependency graph* for $\mathcal{A}$
  - context variables X are vertices of $\mathcal{G}_{\mathcal{A}}$
  - $\mathcal{G}_{\mathcal{A}}$ has an edge (X_i, X_j) if both are arguments of some $A \in \mathcal{A}$

$$A(X) = a(X_1, X_3) b(X_2, X_4) c(X_3, X_4, X_5) d(X_4, X_5) e(X_6, X_7)$$



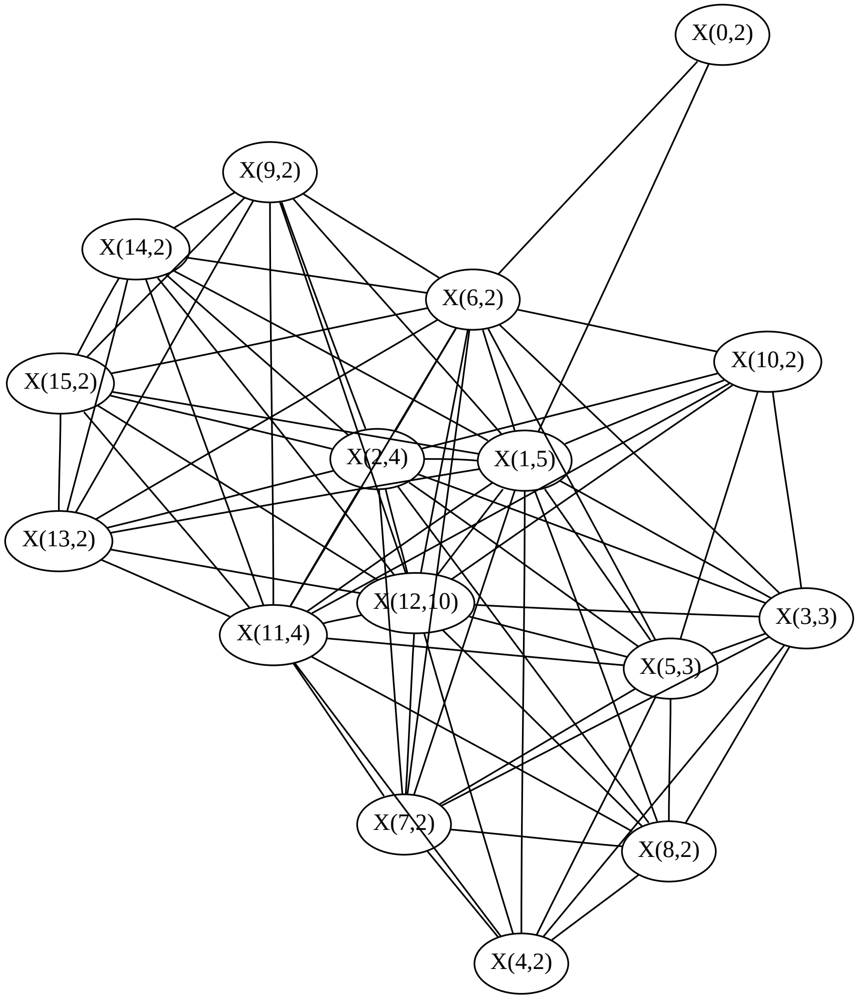
Graphical model computations

$$
\begin{aligned}
Z &= \textstyle\sum_{x \in \mathcal{X}} a(x_1, x_3) b(x_2, x_4) c(x_3, x_4, x_5) d(x_4, x_5) e(x_6, x_7) \\
F_1(X_3) &= \textstyle\sum_{x_1 \in \mathcal{X}_1} a(x_1, X_3) \\
F_2(X_4) &= \textstyle\sum_{x_2 \in \mathcal{X}_2} b(x_2, X_4) \\
F_3(X_4, X_5) &= \textstyle\sum_{x_3 \in \mathcal{X}_3} c(x_3, X_4, X_5) F_1(x_3) \\
F_4(X_5) &= \textstyle\sum_{x_4 \in \mathcal{X}_4} d(x_4, X_5) F_2(x_4) F_3(x_4, X_5) \\
F_5 &= \textstyle\sum_{x_5 \in \mathcal{X}_5} F_4(x_5) \\
F_6(X_7) &= \textstyle\sum_{x_6 \in \mathcal{X}_6} e(x_6, X_7) \\
F_7 &= \textstyle\sum_{x_7 \in \mathcal{X}_7} F_6(x_7) \\
Z &= F_5 F_7
\end{aligned}
$$



Graphical model for Homecentre example

Use a damp, lint-free cloth to wipe the dust and dirt buildup from the scanner plastic window and rollers.



Computational complexity

- Polynomial in m = the *maximum number of conditioning variables* $\geq$ the number of variables in any function A
- m depends on the ordering of variables (and $\mathcal{G}$)
- Finding the variable ordering that minimizes m is NP-complete, but there are good heuristics

Conclusion

- It is possible to compute the statistics needed for parsing and estimation from Maxwell and Kaplan packed parses
  - Generalizes to all Truth Maintenance Systems (not LFG specific)
- Features must be local to parse fragments
  - May require adding features to the grammar
- Computational complexity is polynomial in the number of connected variables
- Makes available techniques for graphical models to packed parse representations
  - Importance sampling
  - Best-first parsing

Future directions

- Can we build a broad-coverage SUBG?
- Reformulate "hard" UFG constraints as "soft" stochastic features
  - Underlying UBG permits all possible structural combinations
  - Grammatical constraints are expressed as stochastic features
- Is the computation tractable if we do this?
- For what tasks is the result significantly better than simpler methods?