Performance Evaluation of Approximation Algorithms for Multipoint Relay Selection

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Abstract—In Mobile Ad Hoc Networks (MANET), the selection of Multipoint Relays provides an efficient routing scheme for efficient broadcast and shortest-path unicast. As such a selection is NP-hard, a heuristic has been designed and effectively implemented in protocols for MANET such as the Optimized Link State Routing protocol (OLSR).

In this paper, we introduce other heuristics that consider the impact of collision by exploiting the topological properties of the network (without assuming a knowledge of geographic positions or geometric properties). For each heuristic, we give its respective provable guaranteed approximation performance when compared to a solution of optimal value.

We compare these dedicated heuristics using several simulations and show that the MPR selection heuristic currently used in OLSR remains competitive and robust against collisions, although the strategies introduced here are effective to prevent the loss of packets.

I. INTRODUCTION

The proliferation of wireless communicating devices has created a wealth of opportunities for the field of mobile computing. In the extreme case of self-organising networks such as Mobile Ad Hoc Networks (MANET), it is still challenging to guarantee efficient communications when mobile nodes roam at will. As the topology changes arbitrarily and rapidly, the communication protocol must obtain dynamically the adequate routing.

MANET are unlike the well-studied cellular systems that rely heavily on the robust structure of the physically connected stations. They are self-organising entities that must distributely choose how to interconnect in order to facilitate the communication within the network. This feature makes them attractive but increases the difficulty of the routing. A mobile node has to cooperate with other hosts to find routes and relay messages. As mobile nodes have a limited communication range, each message may “hop” several times from node to node before reaching its destination (i.e., multihop).

The Internet Engineering Task Force (IETF) - a large open international community of network designers, operators, vendors, and researchers concerned with the evolution of the Internet architecture and the smooth operation of the Internet - is addressing the design of protocols in its MANET working group. Based on the strategy used, MANET routing protocols are generally categorized either as proactive or as reactive, but hybrid protocols exist. Proactive protocols maintain up-to-date routing information by periodically propagating updates throughout the network: e.g., Optimized Link State Routing (OLSR) and Topology Broadcast based on Reverse-Path Forwarding (TBRPF). Reactive protocols create routes only when desired by the source node: e.g., Dynamic Source Routing (DSR) and Ad Hoc On Demand Distance Vector Routing (AODV). Hybrid protocols combine both approaches: e.g., Zone Routing Protocol (ZRP) where each node proactively maintains routes only within a local region.

The availability of fast, cheap and off-the-shelf based equipments that use standards such as IEEE 802.11b, Bluetooth or Hiperlan-2, has opened the possibility to interconnect mobile wireless nodes arbitrarily. Due to its technical specificities, 802.11b is currently widely used as MAC and physical layer below the routing MANET protocols of the IETF.

The concept of Multipoint Relay (MPR) was first introduced in the intra-forwarding protocol in HIPERLAN type 1 standard [11]. It was successfully extended to the case of infrastructureless multihop wireless networks such as MANET. This has been effectively implemented in protocols for Mobile Ad Hoc Networks such as the Optimized Link State Routing protocol (OLSR) [1], [13] where the selection of Multipoint Relays provides effi-
cient routing schemes. It is highly efficient for dense networks. In particular, it provides shortest-path routes for unicast, minimises the flooding of broadcast messages and reduces drastically the overhead of control traffic. Recently, it was shown that the MPRs concept can also be used effectively for reactive MANET protocols [2] and a new IETF-draft for Multipoint Relay flooding in MANET has been submitted [4].

In this paper, we introduce other heuristics to consider the impact of collision by exploiting the topological properties of the network (without assuming a knowledge of geographic positions or geometric properties). For each heuristic, we give its respective guaranteed approximation performance (i.e., when compared to a solution of optimal value) and evaluate their performances.

For completeness, we first recall some of the known complexity results for MPR in Section II and introduce some notations and properties of interest for the other sections. In Section III we introduce several heuristics and give their respective guaranteed approximation performances compared to an optimal algorithm. Finally, in Section IV we compare these dedicated heuristics using several simulations and show that the MPR selection heuristic currently used in OLSR remains competitive and robust when collisions are considered.

II. MPRs Selection and Complexity.

A. MPR definition and OLSR

To introduce and illustrate the importance of the Multipoint Relay concept, we briefly present its use in the OLSR protocol for MANET. We limit our presentation of OLSR to the basic algorithmic point-of-view for the approximation performances, readers interested in further details should read the latest protocol version [1]. Again, MPRs are of interest for many other applications (e.g., [2], [11]).

The goal of Multipoint Relays is to reduce the flooding of broadcast packets in the network by minimizing the duplicate retransmissions locally. Each node selects a subset of neighbors called Multipoint Relays (MPRs) to retransmit broadcast packets. This allows neighbors which are not in the MPR set to read the message without retransmitting it, this prevents the flooding of the network (i.e., the so-called broadcast storm). Of course, each node must select an MPR set among its neighbors that guarantees that all two-hop away nodes will get the packets, i.e., all two-hop away nodes must be a neighbor of a node in the MPR set (see Figure 1 where MPR nodes are in red/grid).

In the OLSR protocol, each node periodically broadcast the information about its immediate neighbors which have selected it as an MPR. Upon receipt of this information, each node calculates and updates its routes to each destination (i.e., the sequence of hops through the successive MPRs from source to destination). Notice that the neighbor discovery overhead is unchanged, thus this process is local and easy to implement distributedly in an efficient way. The MPR broadcasting still follows a simple rule: a node retransmits a broadcast packet if and only if it was received the first time from a node for which it is an MPR. The main gain obtained by introducing MPRs is that: the smaller the MPRs set, the smaller the number of packet retransmissions.

For example, in Figure 1 only three out of the ten neighbors of the source may retransmit the packets, and this MPR set is of minimum size.

Fig. 1. MPR Selection.

Several important properties can be proved about this scheme [11]. In particular, the use of MPRs (instead of all the neighbours) does not destroy the connectivity properties of the network and MPRs provide shortest-path routes for unicast with respect to the original graph.

B. Formal MPR definition and NP-completeness

Notations. We introduce several notations. (For other basic graph-theoretical definitions we refer the reader to graph text books).

Formally we can define the wireless network as a bidirectional undirected graph $G(V, E)$. Nodes $i$ and $j$ share a bidirectional link $(i, j)$ (i.e., an edge) if and only if nodes $i$ and $j$ hear each other and can communicate.
Note that this assumption is not a convenient modeling simplification as, technically, bidirectional links are used to achieve unicast transmission with 802.11b.

Let \( N(u) \) be the neighbors of node \( u \). Let \( N^2(u) \) denote the two-hop neighbors of \( u \) (the neighboring nodes of the neighbors of \( u \) which are not already neighbors of \( u \)). Let \( \Delta \) denote the maximum degree of a node in the graph (i.e., \( \Delta = \max_{u \in V} |N(u)| \)).

Let \( d^+_u(v) = |\{ w \in N(v) | v \in N(u) \text{ and } w \in N^2(u) \}| \), that is the number of neighbors of a neighbor \( v \) of \( u \) that are two-hop away from \( u \). Conversely, let \( d^-_u(w) = |\{ v \in N(w) | v \in N(u) \text{ and } w \in N^2(u) \}| \), that is the number of neighbors of a two-hop neighbor \( w \) of \( u \) that are also neighbors of \( u \). For example in Figure 1, \( d^+_{source}(1) = 8 \) and \( d^-_{source}(q) = 2 \).

Let \( \Delta^+_u \) denote \( \max_{v \in N(u)} d^+_u(v) \) and \( \Delta^-_u \) denote \( \max_{w \in N^2(u)} d^-_u(w) \) (in Figure 1 \( \Delta^+_u = 8 \) and \( \Delta^-_u = 3 \)).

We can now define formally an MPR set of a node \( u \) is a subset \( MPR(u) \) of \( N(u) \) such that:

\[
\forall w \in N^2(u), \exists v \in MPR(u) \text{ such that } w \in N(v).
\]

Let \( MPR^+(u) \) denote an MPR set of minimum cardinality for a node \( u \).

**Definition 1:** Minimum Multipoint Relay (MPR) is defined as:

**Instance:** A network \( G \) (defined as a graph \( G(V,E) \)), a node \( u \) of \( V(G) \) and an integer \( B \).

**Question:** Is there a Multipoint Relay \( MPR(u) \) set of \( u \) of size less than \( B \)?

It is easy to see that this MPR problem is essentially the same as the Minimum Set Cover problem, well-known to be NP-complete (e.g., [8]), and define as follows:

**Definition 2:** Minimum Set Cover (SC) is defined as:

**Instance:** A Collection \( C \) of subsets of a finite set \( S \) and an integer \( B \).

**Question:** Is there a set cover for \( S \), i.e., a subset \( C' \subseteq C \) such that every element in \( S \) belongs to at least one member of \( C' \), such that \( |C'| \leq B \)?

To see the equivalence of the MPR problem with the SC problem, define \( S = N^2(u) \) and assign a subset in \( C \) with each neighbor of \( u \): \( C = \{ S_v | v \in N(u), S_v = \{ w | w \in N^2(u) \text{ and } w \in N(v) \} \} \). Using this polynomial reduction, the MPR problem is equivalent to the SC problem, and hence is NP-complete.

**C. MPR selection in OLSR and approximation bounds**

Although the MPR selection is NP-complete, it is easy to design a heuristic with good guaranteed approximation performances (i.e., when compared to a solution of optimal value). The MPR heuristic currently used in the OLSR implementation [13] follows a “degree-greedy” strategy that selects neighbors that have the largest remaining cover of uncovered two-hop nodes and is described in Figure 2.

1. Select as MPR all the neighbors of \( u \) that are the only neighbors of a 2-hop node from \( u \);
2. While it remains an uncovered 2-hop node from \( u \): select as MPR a neighbor of \( u \) that is neighbor to the largest number of uncovered 2-hop nodes.
   (If ties exist, select a node \( v \) with largest \( d^+_u(v) \).)
3. Discard any MPR node \( v \) such that the MPR set excluding \( v \) covers all 2-hop nodes.

**Fig. 2.** MPR Selection Heuristic used in OLSR.

The first phase is added to adequately use the fact that, whatever the strategy chosen, one-hop nodes that are the sole “cover” of two-hop nodes must be included in the MPR set. For example, in Figure 1 node \( o \) has only one neighbor among the neighbors of the source; hence this respective neighbor (node 1) must be included in the MPR set. The last phase is a simple optimisation check.

Although designed independently [13], this algorithm is the same as the SC algorithm introduced in [12] (except for the addition of the first and last phases of the algorithm that yields better-in-practice solutions without weakening the approximation bound [18]).

Adapting the known approximations bounds of SC [12] for the MPR wireless context considered here, we can immediately deduce the well-known complexity result.

**Theorem 3:** The OLSR heuristic selects an MPR set with the respective performance approximation ratio:

1. \( 1 + \ln |S| = 1 + \ln |N^2(u)| \) in the general case and,
2. \( 1 + \ln \Delta^+_u \), when \( \Delta^+_u \) (the maximum number of two-hop nodes that each one-hop neighbors of \( u \) may cover) is bounded by a constant independent of the size of the network.

It is worth mentioning that this is the best one can hope for in the general case as it is known that SC is not approximable within \( (1 - \varepsilon) \ln |S| \) for any \( \varepsilon > 0 \), unless \( \text{NP} \subseteq \text{DTIME}(n^{O(1/\varepsilon)}) \) [7].

The second bound comes from the fact that when the size of each subset of \( C \) is bounded by a constant \( \Delta \) independent of the size of the input, SC is approximable within \( H(\Delta) = \sum_{j=1}^{\Delta} (1/j) \) (the Harmonic function). This ratio is slightly less than \( 1 + \ln \Delta \) (as \( H(\Delta) < 1 + \ln \Delta < H(\Delta) + \frac{1}{2} \)). This is an important case. Whatever
the wireless technology used, the number of neighbours with which each node can “communicate” is upper bounded by a constant. Indeed, although theoretically the maximum degree of a wireless node could depend on the size of the network, a seminal result on the capacity of wireless networks [10] shows that it will remain small in practice. Other factors like collisions, hidden terminal problem, power saving requirements are also limiting the dimensioning of the network. In fact, with the current technology, the maximum degree, that reaches thousands in theory, can only reach few dozens in practice. Hence, the approximation factors will remain small (i.e., $1 + \ln \Delta$ will be at most 8). This approximation factor compares favorably with known polynomial-time MPR algorithms that achieve a constant approximation-factor for disk-unit graph models (i.e., when each node knows that they have stable bidirectional links with all nodes within given range [5]).

We will exploit such an important topological property for the heuristics described in the next section.

III. ALTERNATE HEURISTICS FOR MPR SELECTION.

One must keep in mind that we are concerned with wireless networks and these often offer some particular topologies as, due to the radio nature of their communications, they often depend on specific physical constraints (to guarantee connectedness for example). In addition to routing, mobile networking differs substantially from traditional networking in many areas such as: scalability (the number of mobile nodes may be huge and continuously varies as some appear or disappear at any time); connectivity (availability and bandwidth); security (increase in physical threats); autonomy (limited power); collisions (due to link interference). For these reasons, it is important to develop routing algorithms that take as much consideration as possible of such difficulties. It is known now that each of these difficulties, even singled out, generate NP-hard problems in wireless environments, while they are polynomially-solvable in wired networks (e.g., bandwidth reservation for Quality of Service [9]).

A simple method to design heuristics for such NP-complete problems is to combine all desired components into weights that represents the trade-offs that seem appropriate. Another possibility is to modify the MPRs selection objective altogether by considering the “quality” of the solution instead of its simplest “quantity”.

In this section, we study the impact of collisions on the MPR selection problem and introduce new heuristics for these specific problems. We exploit properties that are widely available for any wireless networks, such as the maximum degree of each node. We distinguish each heuristic methodologies and its respective complexity.

A. Weighted set cover

Introducing weights as parameters is a most common method used to solve intricate optimisation problems. For example, to maximise the remaining bandwidth at each node at any time while maintaining relatively short paths, the MPRs selection can be done by assigning weights $w(i)$ at each node $i$ that correspond to the bandwidth available at each node. In this case, a variant of the original MPR algorithm presented in Figure 2 could be to modify the last phase to pick the node $v$ which maximized its cover of nodes with available bandwidth.

2. While it remains an uncovered 2-hop nodes from $u$: select as MPR a neighbor of $u$ that has the largest uncovered 2-hop nodes over available bandwidth.

Fig. 3. MPR Selection Heuristic with weight.

The weighted version of the MPR selection algorithm presented previously is also equivalent to the weighted version of the $SC$ problem where the objective is to minimize the sum of the weights in a set cover (see Figure 5). It suffices to change the maximum criteria to $\frac{\Delta^+}{w}$ and to update the weight in each node accordingly. It is also approximable within $1 + \ln |S|$ and within $1 + \ln |\Delta|$ when the size of each subset of $C$ is bounded by a constant $\Delta$ independent of the size of the input [6]. It is important to note that, in both cases, the approximation ratio is not related to the weight function used. In the wireless case at hand, this means that the approximation ratio is only related to $N^2(u)$ and $\Delta^+_u$.

However the quality of the weighted solutions obtained will depend considerably on the adequacy of the weight function used to combine the required features for each problems. A trade-off compromising too much for each problem may generate inadequate solutions for all problems. Assigning weights that genuinely represent the trade-off of the combined problems to solve is not always easy. In particular, when considering two problems, one objective is to be maximised whilst the other is to be minimised.

B. In-degree greedy set cover

The original MPR selection complexity mainly depends of the out-degree of the neighbour nodes $v$ of the
source \( u \), i.e., the maximum possible value of \( d_u^+(v) \), if this value is bounded. Due to their intrinsic connectedness, wireless networks may be dense and highly clustered. In this case, one may observe that conversely, \( \Delta_u^- \), the maximum value of the in-degree \( d_u^-(w) \) of the two-hop nodes \( w \in N^2(u) \), is likely to be a smaller constant.

This observation allows us to introduce a variant of the original MPR selection heuristic in Figure 4 where \( MPR^- (u) \) denotes the MPRs set returned by the algorithm.

2. While it exists an uncovered node \( w \) 2-hop from \( u \): select as MPR a neighbor of \( w \) that is neighbor to the smallest number of uncovered 2-hop nodes.

Fig. 4. MPR Selection Heuristic using In-Degree.

We give the approximation performance of this new algorithm.

**Theorem 4:** When \( \Delta_u^- \) is bounded by a constant independent of the size of the input, the MPR Selection algorithm using in-degree presented in Figure 4 guarantees an approximation ratio to the optimal size of the MPR set of \( \Delta_u^- \) for a source node \( u \).

**Proof:** The algorithm introduced here follows the same line as the algorithm presented in [3], hence the same bound and the same proof apply. As for OLSR, the major difference is the introduction of an initial and final phases that yields better-in-practice solutions without weakening the approximation bound.

A weighted version of this algorithm where each node \( i \) is assigned a weight \( \text{weight}(i) \) also exists. Again this algorithm has the same approximation ratio which is independent of the weight function used. Its proof follows the same lines than in [3].

It is clear that if \( \Delta_u^- \) is large or larger than \( 1 + \ln \Delta_u^+ \), this algorithm is inferior to the original MPR algorithm implemented in the OLSR protocol. However, as both heuristics have a small polynomial complexity, it is reasonable to run both in order to choose the best MPR set and guarantee an approximation ratio to the optimal weight of the MPR set of \( u \) of: \( \min(\Delta_u^-, 1 + \ln \Delta_u^+) \).

C. MPR Selection with Minimum Overlapping.

The aim of the MPRs during the broadcast phase is to forward packets effectively by reducing redundancy in order to limit the traffic and risk of collisions. In Figure 4 the size of MPRs set is minimum, however nodes 1 and 4 both cover nodes 1 and 5. Such overlap will be detrimental to some applications as known problems such as the so-called Hidden Terminal may occur and packets may be lost.

A possible strategy is to spread as evenly as possible the MPR nodes around the source to reduce the overlapping of two-hop nodes cover.

Ideally, one would like to obtain small MPR set for which MPR nodes “cover” disjoint sets of two-hop away nodes. Of course, this may be impossible to achieve if two neighbors of the source are each the sole cover of a respective two-hop node and overlap in their covers. This unfortunate case may involve an arbitrarily large number of nodes (up to the maximum degree of the nodes). Hence, it is pointless to expect to reduce the maximum number of overlapping per node. However, it is possible to limit the impact of the overall overlapping according to a given source, i.e., the least overall overlapping of the MPR set. Using the same topology as Figure 4 one could choose a MPR set that reduces the number of nodes that “overlap” from 3 to 1, by incrementing the size of the MPR set (see Figure 5).

This minimization problem is known as the Minimum Exact Cover. It is NP-complete and is defined formally as follows.

**Definition 5:** Minimum Exact Cover (EC) is defined as:

**Instance:** A Collection \( C \) of subsets of a finite set \( S \) and an integer \( B \).

**Question:** Is there an exact cover for \( S \), i.e., a subset \( C' \subseteq C \) such that every element in \( S \) belongs to at least one member of \( C' \), such that

\[
\sum_{c \in C'} |c| \leq B
\]
This is approximable within $1 + \ln |S|$ [12] but as hard to approximate as $SC$ [14]. The only difference between $SC$ and $EC$ is the definition of the objective function. Again the same approximation ratio is achievable by the weighted version of this problem (where the objective is to minimize the sum of the weights in the set cover and where the weights are counted as many times as that they are covered).

Intrinsically, it is not a slight formal modification. Although related, the two problems $EC$ and $SC$ may diverge. The best approximation heuristic known for $SC$ (as is the MPR selection algorithm presented in Figure 2) may obtain an optimal solution for a given instance of the $SC$ problem while obtaining an extremely poor solution for the same given instance for the $EC$ problem: the approximation factor could be as bad as $O(|S|)$.

Fortunately a heuristic with good approximation ratio can be designed. The idea of the algorithm is to limit the overlapping by greedily selecting nodes that minimize their respective ratio of the number of already covered nodes over the number of uncovered nodes.

For completeness, we present the specific heuristic for the MPRs selection with reduced overlap in Figure 6. We highlighted the changes compared with the algorithm in Figure 2.

2. While it remains an uncovered 2-hop node from $u$: select as MPR a neighbor of $u$ that has the smallest covered over uncovered 2-hop nodes ratio.

Fig. 6. MPR Selection Heuristic for minimum overlapping.

We can immediately deduced the two following theorems.

**Theorem 6:** The MPR Selection Algorithm for limited overlapping presented in Figure 6 guarantees an approximation ratio to the optimal size of the $EC$ MPR set of $1 + \ln |N^2(u)|$ for a source node $u$.

**Theorem 7:** When $\Delta^+_u$ is bounded by a constant independent of the size of the input, the MPR Selection Algorithm for limited cover overlapping presented in Figure 6 guarantees an approximation ratio of $1 + \ln \Delta^+_u$ for a source node $u$.

**Proof:** The algorithm introduced here follows the same lines as the algorithm presented in Section 6 in [12], hence the same bound and the same proof apply. As for OLSR, the major difference is the introduction of an initial and final phases that yields better-in-practice solutions without weakening the approximation bound.

Similarly, in each case, it is known that one can build a graph for which such a bound is attainable [12].

### D. MPR Selection with secondary priority.

A simple way to tackle several problems is to consider them one after the other. This is actually what is currently implemented in OLSR, as described earlier in Figure 2. In case of ties, priority is given to the node with maximum number of neighbours in $N^2$. Although it is clear that this method explicitly gives a priority to the routing problem, this strategy may maximise the possible overlap by MPR nodes as opposed to the previous $EC$ previous strategy.

For the sake of the comparison, we introduce two priority variants in case of multiple nodes with largest number of uncovered two-hop nodes: one which selects a node with minimum number of neighbours in $N^2$ and another which selects randomly. The algorithms are formally described in Figure 7 and in Figure 8 respectively.

2. While it remains an uncovered 2-hop node from $u$: select as MPR a neighbor of $u$ that is neighbor to the largest number of uncovered 2-hop nodes. (If ties exist, select the node with smallest $d^+_u$.)

Fig. 7. MPR Selection Heuristic with prioritized overlap.

2. While it remains an uncovered 2-hop node from $u$: select as MPR a neighbor of $u$ that is neighbor to the largest number of uncovered 2-hop nodes. (If ties exist, pick randomly among these nodes.)

Fig. 8. MPR Selection Heuristic prioritised randomly.

As the priority is only triggered in the case of ties, the following corollary follows.

**Corollary 8:** The Priority heuristics select an MPR set with the same performance approximation ratio than the OLSR heuristic given in Theorem 3.

### E. Impact of the geometric model

In this paper, the heuristics considered do not assume a knowledge of positions (e.g., using either GPS or sensors) or geometric properties such as a unit-disk graph model (i.e., where an edge exists between two nodes if and only of their Euclidean distance is less or equal to a given constant). However, for simplicity, we used this model for our experimental simulations.
We remark that, in such a model, the OLSR MPR selection algorithm is likely to perform better than the MPR selection using in-degree. Indeed, by comparing $\Delta_n^+$ and $\Delta_n^-$ in the unit-disk graph, it is easy to see that the area covered by a neighbor of the source can be less than twice the area that a two-hop node may cover within the neighboring of the source.

Such an extreme case occurs when a two-hop node is as close as possible from the source’s range as depicted in Figure 9 where the source node $u$ is in the top unit-disk and a two-hop node $w$ is in the bottom unit-disk (and the $A$ region). On one hand, the maximum area including the source’s neighbors that $w$ can cover corresponds to the region $B$ represented by vertical lines, and the number of nodes in the area $B$ will define the maximum in-degree $d^-_u(w)$ of a two-hop node. On the other hand, a source’s neighbor may cover a region as large as the $A$ region (the grey region), and the number of nodes in the area $A$ will define the maximum out-degree $d^+_u(v)$ of a neighbor $v$ of the source in $N^2(u)$. Let us define $D$ as a whole unit-disk area. Note that $A + B$ covers exactly a whole unit-disk area $D$. By defining in the figure an area $C$ (represented by the area with horizontal lines) and standard geometry, it is easy to see that $D = 3(B - C)$ and $A = 2B - 3C$. Thus, in the extreme case $B > \frac{1}{2}A$, the minimum value in $\min(\Delta_n^-, 1 + \ln \Delta_n^+)$ is likely to be $(1 + \ln \Delta_n^+)$ if nodes are uniformly distributed at random.

![Fig. 9. Maximum cover for two-hop nodes.](image)

IV. Simulations and Analysis.

In this section, we compare the performance of the OLSR selection heuristic with the four new heuristics using several simulations.

A. Simulator design

In this paper, we only described the simulator features that are relevant for this study. Further details on other features (e.g., visual interface) can be found in [17].

In this context, MANET are dynamically and arbitrarily created wireless networks. Possible collisions, collision avoidance mechanisms, and queuing are also modelled. The specification for the underlying MAC and PHY layer is assumed to be in accordance with 802.11b specifications. Mobility model (for both node mobility and changes in propagation medium) is however not modelled in this version of the simulator.

The network is modelled by a random unit-disk graph, i.e., there exists a direct link between any two nodes in this network if and only if the distance between the two nodes is less than or equal to the radio range $r$. Hence, all the links between the nodes are considered to be bidirectional. The network consists of $n$ nodes distributed uniformly at random in a network area of length $L$ and width $W$. We also make a simple remark in respect to this model in subsection 4.4.

Only flooding of packets is simulated here (i.e., there is no acknowledgement of reception, nor retransmission when error of reception occurs). Each node in the network is capable of calculating and storing its MPR set according to a chosen algorithm. It also stores a list of its immediate neighbours, and a list of nodes that has selected it as an MPR (MPR selector).

**Scheduler.** In a dense network, packets may be lost due to interference and collisions. As there might be some delays (propagation, jitter, backoff, etc), a simple scheduler was designed. A number of time slots are initially declared. The slots do not assume a slotted system but just simulate some (discrete) time when the packet are sent. The transmission of a single packet can take $k$ slots, where $k$ is constant for each experiment. This models the case of overlapping transmission. In this paper, we take $k = 1$.

The node that starts a broadcast starts at slot 0. It sends the packets to all of its neighbours. Each of the receivers processes the packet. Depending on the routing protocol chosen, if it needs to forward the packet (for example in OLSR, if it received it for the first time, and from an MPR selector), it chooses a random slot number. This receiver is then added to that slot as a transmitter. The scheduler checks at each slot for a list of transmitters, and calculates the possible receptors for them. If a node appears more than once in the list of receptors, it is assumed that the node should receive packets from two nodes at the same time resulting in a collision and loss of packets. In this case, the node is removed from the list of receptors. The packet is transmitted to the remaining receptors, which again repeat the random slot selection process. The broadcast completes when all the slots are empty. This allows to simulate the possible case when the broadcast failed to get the packet to all nodes.
**Settings used.** Various sized networks with different parameters were used to test the MPR algorithms. Between 50 and 400 nodes were randomly placed in a square field of 1000m by 1000m, each node with a range of 200 meters. For each simulation, each of the nodes in the random network initiates a broadcast once (one after the other). Ten simulations with identical parameters are made, and the average of the results are presented here.

**B. Transmissions, receptions and MPR count**

![Graph](image1)

**Fig. 10.** Overall number of retransmissions (without collisions).

To show the interest of all heuristics, pure flooding (i.e., every node retransmits the broadcast packets if and only if it received it for the first time) was also simulated for comparison of the gain in number of retransmissions (i.e., the total number of packets sent and forwarded by the nodes in the network). First, we assumed that the conditions were ideal and there were no collisions. Results are presented in Figure 10.

As expected, MPR schemes prevent a large amount of redundant transmission as many nodes are relieved from their responsibility to retransmit broadcast packets from some of their neighbours.

As a first attempt at comparing all five heuristics, we measured the average MPR count per node, the average number of retransmissions, and the average number of receptions, for each broadcast test case (see Figure 11).

The number of MPR nodes that all new presented algorithms was either the same as in the case original MPR selection algorithm or increased marginally on average (i.e., by at most 1 or 2 for some nodes) despite the increase in density of nodes in the network. This can be easily explained by the fact that the initial phase of the algorithm (picking MPR nodes that are the sole neighbour of a two-hop node) induces up to 68% of all MPR. Thus, the MPR selection for each heuristic may only vary among the remaining 32% of MPRs.

However, the number of retransmissions increased for the case of the new MPR selection algorithm as compared to the original MPR selection algorithm. In particular, the In-degree algorithm was found to increase the retransmissions by up to 55%, and hence was not included in further evaluations. An analysis of this poor performance is given in the subsection III-E. In case of minimum overlap algorithm, the number of retransmis-

![Graph](image2)

**Fig. 11.** MPR per node, Retransmissions and Receptions.
sions was found to increase by 11% to 19% on average. As expected for the prioritised and random algorithms, the number of retransmissions was sometimes slightly smaller than the original algorithm, but in average, it was found to be always between 2% and 4%.

This difference between the average numbers of retransmissions has a direct impact on the number of receptions that occurred (including those discarded if they were not received for the first time or not from an MPR selector). Since one more MPR node usually means more transmissions by the MPR nodes of this new MPR node, the number of receptions increased in all new heuristics. However, in case of min-overlap algorithm the overall number of receptions only increased by 7% to 12% (that is less than the 11-19% increase of the number of retransmissions), suggesting a positive impact of the overlapping strategy.

The most interesting result is the occasional reduction in number of receptions for the prioritised overlap (despite the retransmission increase) by up to 2%. The occasional increase never passes 1.5%. Similarly, the overall number of receptions only increased by 0.11% to 1.6% in case of the prioritised random algorithms.

C. Scheduling and collisions

All MPR-based algorithms are compromising between reliability (i.e., the packet will arrive in every node) and redundancy (some nodes will receive packet more than once).

The introduction of scheduling in the simulator created a more realistic environment, where the nodes contend for network resources, and so there might be delay between reception of a message and its forwarding, and there is possibility of collision of packets. The scheduler helps in assessing the impact of collision for reliability and redundancy. The number of slots tested were 10, 16 and 24. Plotting results are presented in Figures 13 and 14 respectively.

It was seen that, for the prioritized heuristics, the number of collisions in the network on average were similar to the OLSR heuristic, despite the expectation that the new algorithm will introduce less collisions (see Table I). (In the best case, the collision counts decreased by less than 0.5% for the new prioritised heuristics, and increased by at 0.3% in the worst case, which is marginal.) As expected, when the slot size was increased from 10 to 24, the average number of collision per broadcast decreased for each algorithm, as fewer slots mean more collisions. Overall the Min-Overlap collision count increases by 16 to 18%. These results can be explained by the fact that these schemes only prevent collisions that are close to the source (i.e., local collisions involving MPRs of the same node). The global impact remains difficult to control.

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>10</th>
<th>16</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>0.31</td>
<td>-0.08</td>
<td>0.00</td>
</tr>
<tr>
<td>Prioritised</td>
<td>0.32</td>
<td>-0.52</td>
<td>-0.10</td>
</tr>
<tr>
<td>MinOverlap</td>
<td>16.76</td>
<td>16.82</td>
<td>17.66</td>
</tr>
</tbody>
</table>

TABLE I
RATIO OF COLLISIONS COMPARED TO OLSR.

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>10</th>
<th>16</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>1.40</td>
<td>1.36</td>
<td>1.41</td>
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<tr>
<td>Prioritised</td>
<td>3.21</td>
<td>3.21</td>
<td>3.30</td>
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<tr>
<td>MinOverlap</td>
<td>15.11</td>
<td>15.19</td>
<td>15.43</td>
</tr>
</tbody>
</table>

TABLE II
RATIO OF TRANSMISSION AND RECEPTION INCREASE.

This was confirmed by counting the retransmissions and receptions (see Table III for average). For retransmissions, the new algorithm resulted in either the same number of retransmissions or an increase by around 1.5% for random and 3.3% for prioritised overlap. Min overlap about 15%.

The number of receptions that occurred also increased slightly by 0.6% for the random priority. It was thus noticed that the slight reduction, that occurred in the case of non-scheduled receptions for the prioritised algorithms, did not occur in case of the scheduled simulation.

Reliability. We also measured the number of nodes that did not manage to get any packet at the end of each broadcast due to collisions (see Figure 12). Average results are presented in Table III. The minimum Overlapping heuristic (based on the Exact Set Cover strategy) improves the reliability of each broadcast compared to all other heuristics (based on Set Cover strategy) despite generating more collisions and more receptions. It should be emphasized that the fact that OLSR heuristic is the weakest compared to other selection heuristics does not

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>10</th>
<th>16</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLSR</td>
<td>2.14</td>
<td>1.17</td>
<td>0.70</td>
</tr>
<tr>
<td>Random</td>
<td>2.07</td>
<td>1.10</td>
<td>0.71</td>
</tr>
<tr>
<td>Prioritised</td>
<td>2.06</td>
<td>1.04</td>
<td>0.65</td>
</tr>
<tr>
<td>MinOverlap</td>
<td>1.52</td>
<td>0.76</td>
<td>0.43</td>
</tr>
</tbody>
</table>

TABLE III
NUMBER OF NODES THAT DID NOT RECEIVED PACKET.
jeopardise the OLSR protocol as the nodes missing out differ for each broadcast.

V. CONCLUSION

In this paper, we have recalled some important properties of the known approximation algorithms for computing a MPR set of minimum cardinality (when no knowledge of the geographic locations or of geometric properties are assumed). We have introduced several variants that may be of interest when other optimizations are considered concurrently. All these variants extend immediately to weighted versions and have provable approximation performances that solely depend on the maximum degree of the nodes in the network.

When possible collisions are considered, experimental simulations have shown that the MPR selection heuristic currently used in OLSR remains a robust candidate to achieve the desirable trade-off between redundancy (useless retransmissions and receptions) and reliability (reduced loss of packets).

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Fig. 14. Experiments with 16 slots.

Fig. 15. Experiments with 24 slots.

REFERENCES


