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Degenerate Trimble n-categories

Trimble's approach to defining weak n-categories is by iterated enrichment, where each stage of enrichment is parametrised by the action of an operad. For some time it was thought that this definition would be "too strict" because although associativity and units are made weak, interchange remains strict. However, in the intervening years, work by Joyal and Kock [2] has shown that an alternative strictification to the usual one is possible: every weak 3-category is equivalent to one in which everything is strict except units. This has brought Trimble's definition back into play.

The definition relies heavily on a particular reparametrisation operad which is abstractly appealing (it has a good universal property) but unwieldy in practice. In previous work [1] we have investigated what properties Trimble's operad needs in order to make the definition work, with the aim of finding smaller operads that also work. In this talk we will express these properties in the form of explicit self-actions of the operad, and then derive a suitable such operad from the little intervals operad. We then study the doubly degenerate 3-categories parametrised by it. We show how each one gives rise to a braided monoidal category, and moreover, that the free one on terminal data gives rise to braids in 3-space. The construction relies critically on the good properties of the operad we have constructed, and the result indicates that Trimble's definition is weak enough to capture the full weakness of weak 3-categories and homotopy 3-types.

References

[1] Eugenia Cheng and Nick Gurski. Towards an n-category of cobordisms. *Theory and Applications of Categories*, 18:274–302, 2007.

[2] André Joyal and Joachim Kock. Weak units and homotopy 3-types. In Batanin, Davydov, Johnson, Lack, and Neeman, editors, *Categories in Algebra, Geometry and Mathematical Physics, proceedings of Streetfest*, volume 431 of *Contemporary Math.*, pages 257–276. AMS, 2007.