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Skew monoidal structure via the Catalan simplical set

Skew monoidal categories are much like monoidal categories except that their associativity and unit constraints are non-invertible. Recent work by Kornel Szlachányi has related them to bialgebroids and Steve Lack and Ross Street have linked them to quantum categories. The definition of skew monoidal category involves three families of maps $\lambda\colon I.A \to A$, $\rho\colon A \to A.I$, and $\alpha\colon (A.B).C \to A.(B.C)$ together with five axioms (Mac Lane's original five). These axioms don't make all diagrams commute and so we ask "Why these axioms and not others?".

It has been known for some time that associativity constraints (associahedra) can be systematically generated using orientals. It has also been noted by Mike Johnson that the unit constraints can be obtained in a similar fashion. We unite these two observations by constructing a simplicial set $\mathbb{X}$ that contains exactly the combinatorial data required to describe skew monoidality. In particular we show that simplicial maps from $\mathbb{X}$ into a suitably defined nerve of Cat give skew monoidal categories. We also show that $\mathbb{X}$ has a Catalan number of simplices at each dimension and is the nerve of a very simple 2-category.

This is joint work with Richard Garner, Steve Lack and Ross Street.