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Rings and near-rings in a rig category

We define rig categories, which are categories with two monoidal operations, "tensor product", $\diamond$ and "tensor sum", $\star$. Tensor products distribute over tensor sums, via a distributive law

$$\delta : (A \star B) \diamond C \mapsto (A \diamond C) \star (B \diamond C)$$

which is natural in A, B and C, and satisfies several coherence conditions. Rig categories have been studied by Kelly [2] and Laplaza [1], under the name ring categories, and with the additional constraint that δ is an isomorphism. However, we shall look at examples where this is not the case. Our primary examples include presheaves on finite sets, and Joyal's category of species.

Moreover, we define near-ring and ring objects in a rig category $\mathcal{C}$. We study the free ring $\mathcal{S}(M)$ on a monoid M, as well as the free near-ring $\mathcal{F}(M)$, and distributively generated near-ring $\mathcal{T}(M)$, provided the category $\mathcal{C}$ is cocomplete. We then show how these constructions actually come from distributive functors, which is the rig category analogue of a ring homomorphism.

This is joint work with Marcelo Aguiar and Swapneel Mahajan.

References

[1] M. Laplaza, Coherence for distributivity, *Lecture Notes in Mathematics* **281**, Springer Verlag, Berlin, 1972, pp. 29-72.

[2] G. Kelly, Coherence theorems for lax algebras and distributive laws, *Lecture Notes in Mathematics 420*, Springer Verlag, Berlin, 1974, pp. 281-375.