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Grothendieck topologies and Grothendieck quantales

A locale L can be thought of as a particular poset $(L, \leq)$ but also as a particular monoid $(L, \wedge, \top)$. The precise relation between these two manifestations of L is that the *poset* is exactly the category of left adjoints in the split-idempotent completion of the *monoid*. This generalises to Grothendieck topologies: each site determines, and is determined by, a quantaloid (Sup-enriched category) of closed cribles, which in turn is always the split-idempotent completion of a particular quantale (monoid in Sup). The latter are the "Grothendieck quantales" of the title. Besides locales, also inverse quantal frames are examples of this notion. This facilitates the description of sheaf toposes as particular Sup-module categories.

This is joint work with Hans Heymans.