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Homotopy type theory: towards Grothendieck's dream

Several decades ago, Grothendieck proposed that "∞-groupoids" could replace topological spaces as a context for homotopy theory. Higher category theory has since exploded with definitions and applications, but Grothendieck's original intent remains unrealized: the notions of ∞-groupoid amenable to homotopy theory are not the "algebraic" sort he envisioned.

I propose that the recent discovery of "homotopy type theory" realizes Grothendieck's dream in an unexpected way. Rather than *defining* ∞-groupoids and proving an equivalence to classical homotopy theory, homotopy type theory is an *axiomatic* theory of ∞-groupoids, which are "algebraic" like Grothendieck's, but admit a model in classical homotopy theory. Moreover, the new ideas of *univalence* and *higher induction* yield straightforward proofs of homotopy-theoretic facts such as (in the past six months alone) $\pi_n(S^n) = \mathbb{Z}$, van Kampen's theorem, the Freudenthal suspension theorem, the Blakers-Massey theorem, and $\pi_3(S^2) = \mathbb{Z}$.

Homotopy type theory can also serve as a foundational system for mathematics whose basic objects are ∞-groupoids rather than sets. This has pleasant consequences for category theory: for instance, we can give a definition of "category" such that all properties and constructions are automatically invariant under equivalence of categories.

This talk represents joint work of the entire Univalent Foundations Program at IAS.