Daniel Schäppi
University of Chicago

Weakly Tannakian categories

Classical Tannaka duality is a duality between groups and their categories of representations. The two basic questions it answers are the reconstruction problem (when can a group be reconstructed from its category of representations?) and the recognition problem (can we characterize categories of representations abstractly?).

Using the notion of a weakly Tannakian category, we can solve the recognition problem for categories of comodules over flat Hopf algebroids. In the language of algebraic geometry, this provides a recognition principle for categories of coherent sheaves of certain algebraic stacks and algebraic spaces.

Combining this with a result due to Lurie, we can establish an equivalence between a certain 2-category of algebraic stacks and the 2-category of weakly Tannakian categories, right exact symmetric monoidal functors, and symmetric monoidal natural transformations. To illustrate the usefulness of this biequivalence, we show that the category of coherent sheaves on a product of two projective varieties is given by the Deligne tensor product of the categories of coherent sheaves on the two factors.