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Span, cospan, and other double categories

We present the main results of the article "Span, cospan, and other double categories" (TAC 26) together with some new examples. In particular, we show that if any two of the following properties hold, then so does the third, wherein $\mathbb{D}$ is a double category such that $\mathbb{D}_0$ has pushouts:

- $\mathbb{D}$ has companions and conjoints;
- $\mathbb{D}$ has 1-cotabulators, i.e., $\Delta \colon \mathbb{D}_0 \to \mathbb{D}_1$ has a left adjoint;
- There is an oplax/normal lax adjunction from $\mathbb{D}$ to $\mathbb{C}\mathrm{ospan}(\mathbb{D}_0)$ which restricts to the identity on $\mathbb{D}_0$.

There is an analogous dual result involving 1-tabulators and $\mathbb{S}\mathrm{pan}$. Examples include the double categories of posets, small categories, locales, topological spaces, toposes, suplattices, and commutative rings. These seven double categories are easily seen to have companions, conjoints, and 1-cotabulators. All but the last have 1-tabulators. There are also two double categories of commutative quantales, one related to commutative rings and the other to locales. Both have companions and conjoints, as well as 1-cotabulators, The first lacks 1-tabulators, and hence, an adjunction with its span category. The second has 1-tabulators which can be constructed by a variation of that of posets, small categories, locales, topological spaces, and toposes.