

Peter LeFanu Lumsdaine

Institute for Advanced Study

Beck–Chevalley conditions for dependent type theory

Dependent Type Theory is a powerful and congenial basis for logical systems, but the study of its models is plagued by coherence problems. Specifically, strict models of syntax require operations to commute on the nose with pullback functors; in natural settings, many operations commute with pullback only up to isomorphism (the typical *Beck–Chevalley condition*), or even not at all. Coherence theorems bridge this gap, taking categories with logical structure satisfying weak commutation conditions and “improving” them to structures satisfying strict commutation.

I will survey a range of Beck–Chevalley-type conditions, and present a new coherence theorem (joint work with Michael Warren), showing that assuming existence of certain exponentials, one of the weakest conditions one might consider (*weakly stable* logical operations) can be improved to full, strict commutation. The construction can be viewed in terms of *local universes*, and is related on the syntactic side to *explicit* or *delayed substitution*.