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Homotopy theory of algebras of polynomial monads

We study the existence and left properness of transferred model structures for "monoid-like" objects in monoidal model categories. These include genuine monoids, but also all kinds of operads, e.g. symmetric, cyclic, modular, n-operads, dioperads, properads and (wheeled) PROP's. All these structures can be realised as algebras over polynomial monads. To get a useful criterion for left properness we introduce a model-theoretical concept of h-cofibration and call a model category h-monoidal if it is monoidal and if the tensor of a (trivial) cofibration with an object yields a (trivial) h-cofibration. Most of the model categories used in algebraic topology or higher category theory are h-monoidal.

We give a general condition for a polynomial monad which ensures the existence and (relative) left properness of a transferred model structure for its algebras in an h-monoidal model category. This condition is of a combinatorial nature and singles out a special class of polynomial monads which we call tame. Many important polynomial monads are shown to be tame. On the other hand there are interesting polynomial monads which are not tame, for example, the monads for modular operads or PROPs. We show that failure of the tameness condition can be used to find obstructions for the existence of transferred model structure on algebras.

This is joint work with Clemens Berger.