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Bifibrational duality for groups and in general categories

An axiomatic approach in the study of the category of groups was initiated by S. Mac Lane in *Duality for groups* (1950), where the idea of using dual axioms for such a study is proposed. For the category of abelian groups this has subsequently led to the introduction of the notion of an abelian category, defined indeed by dual axioms, while at the same time it became evident that it is impossible to find dual axioms satisfied by the category of groups and powerful enough to deduce results such as isomorphism theorems, homological diagram lemmas and other exactness properties valid for not necessarily abelian groups. From the modern perspective, this can be confirmed by the fact that a semi-abelian category whose dual is also semi-abelian is already an abelian category. In this talk it will be shown that if we replace the structure of a category with a pair, consisting of a category and a faithful bifibration given on it, then, quite unexpectedly, it becomes possible to develop a self-dual approach to groups, by looking at the pair consisting of the category of groups and the bifibration of subobjects in it. Here, duality refers to replacing a bifibration with its dual bifibration over the dual category. We will also show that this approach can be extended from the category of groups to abstract categorical contexts, where, among other things, it reveals new suprising links between different exactness properties studied in categorical algebra.