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The fundamental group functor as a Kan extension

The main aim of this talk is to show that

$$\begin{array}{ccc} & \mathbf{NExt}_\Gamma(\mathcal{C}) & \\ \text{Cod} \swarrow & \xrightarrow{\delta} & \searrow \text{Ker} \\ \mathcal{C} & \xrightarrow{\pi_1(-, I)} & \mathcal{X} \end{array}$$

is a Kan extension. Here Γ is a categorical Galois structure as defined by Janelidze, consisting of an adjunction between the categories \mathcal{C} and \mathcal{X} and certain classes of morphisms, and $\mathbf{NExt}_\Gamma(\mathcal{C})$ is the category of *normal extensions*, which are defined via the Galois structure Γ . In fact, we will show that $\pi_1(-, I)$ is the Kan extension of the Galois group functor $\text{Gal}_\Gamma(-, 0): \mathbf{NExt}_\Gamma(\mathcal{C}) \rightarrow \mathbf{GpX}$ along Cod. This Galois group functor is also defined in the context of categorical Galois theory and lands in the internal groups of the category \mathcal{X} . The talk will explain all the ingredients in these two Kan extensions and will then concentrate on the second one, concluding by showing how the first Kan extension follows from this second one.

This is joint work with Tomas Everaert and Tim Van der Linden.