Richard Garner
Macquarie University

Finitary monads, Lawvere theories, and Cauchy completion

There are two main ways in which a category theorist can articulate the notion of equational theory: using Lawvere theories, and using finitary monads on **Set**. The equivalence between the two formulations is well-known, and well-understood; this talk will reconsider it from the perspective of enriched category theory. On the one hand, a finitary monad on **Set** is a monoid in the category $\mathbf{End}_f(\mathbf{Set})$ of finitary endofunctors of **Set**, so equally a one-object $\mathbf{End}_f(\mathbf{Set})$-enriched category. On the other hand, it turns out that every category with finite powers—and so in particular, every Lawvere theory—bears a canonical $\mathbf{End}_f(\mathbf{Set})$-enrichment. In fact, we find that categories with finite powers are precisely the same thing as $\mathbf{End}_f(\mathbf{Set})$-categories admitting a certain class Φ of absolute colimits, and that, from this perspective, the mechanism by which a Lawvere theory is associated to a particular finitary monad is simply that of Cauchy completion relative to Φ. The enriched-categorical perspective explains also the semantics: models of a Lawvere theory, or algebras of a finitary monad, are simply $\mathbf{End}_f(\mathbf{Set})$-functors from the theory or the monad into the (Φ-cocomplete) $\mathbf{End}_f(\mathbf{Set})$-category of sets, so that the equivalence of the models of the theory with the algebras of the associated monad is a direct consequence of the universal property of Cauchy completion.