Nicholas Duncan

University of Cambridge

Calibrated toposes in geometry

The theory of calibrations on a topos is one manner in which we can study "gros" and "petit" toposes. A calibration is a subclass of morphisms in a topos, for which we think of each morphism as having "discrete fibres". The collection of all morphisms in a calibration with a common codomain A in $\mathcal{E}$ form a subtopos of $\mathcal{E}/A$ which we call the petit topos associated with the object A.

In this talk we will cover the use of calibrations in various gros toposes relating to geometry, such as the Zariski topos of algebraic geometry. Sheaves over the spectrum of a scheme X correspond to morphisms in the calibration with codomain X. Therefore using a calibration allows us to study a scheme using both the locally ringed space viewpoint and the functor of points viewpoint at the same time.

Furthermore, given a geometric morphism we can push forward a calibration from one topos to another. This will allow us to define calibrations on related toposes. The Zariski topos, being the classifying topos of local rings, has the generic local ring as one of its objects. The theory of modules over this local ring also has a classifying topos, so we can take the pushforward of the calibration on the Zariski topos along a suitable geometric morphism to turn this classifying topos into a gros topos, which will allow us to study vector bundle-like objects.