

Thomas Athorne

University of Sheffield

Cell complexes are coalgebras

The usual “small object argument” generates a weak factorisation system from a set of generating maps. A more categorically motivated version of this argument, due to Richard Garner, provides an algebraic weak factorisation system instead. This structure is a functorial factorisation system that can be expressed as a monad and a comonad on an arrow category. Garner conjectured that the coalgebras for this comonad might be relative cell complexes. We formalise this conjecture by defining the category of relative cell complexes generated by a set of maps in any complete and cocomplete category. It can then be proved for a wide class of examples that this is equivalent to the coalgebra category for the algebraic weak factorisation system created using the small object argument. We discuss the conditions for this to hold, and finally point towards applications in higher category theory.