# Trust Transitivity in Complex Social Networks

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## Abstract

In Online Social Networks (OSNs), participants can conduct rich activities, where trust is one of the most important factors for their decision making. This necessitates the evaluation of the trustworthiness between two unknown participants along the social trust paths between them based on the *trust transitivity* properties (i.e., if $A$ trusts $B$ and $B$ trusts $C$, then $A$ can trust $C$ to some extent). In order to compute more reasonable trust value between two unknown participants, a critical and challenging problem is to make clear how and to what extent trust is transitive along a social trust path.

To address this problem, we first propose a new complex social network structure that takes, besides *trust*, *social relationships*, *recommendation roles* and *preference similarity* between participants into account. These factors have significant influence on trust transitivity. We then propose a general concept, called *Quality of Trust Transitivity* (QoTT), that takes any factor with impact on trust transitivity as an attribute to illustrate the ability of a trust path to guarantee a certain level of quality in trust transitivity. Finally, we propose a novel Multiple QoTT Constrained Trust Transitivity (MQCTT) model. The results of our experiments demonstrate that our proposed MQCTT model follows the properties of trust and the principles illustrated in social psychology, and thus can compute more resonable trust values than existing methods that consider neither the impact of social aspects nor the properties of trust.

## Introduction

In recent years, social networking sites have been used as a means for a variety of rich activities. For example, according to a survey on 2600 hiring managers in June 2009 by CareerBuilder[1] (a popular job hunting website), 45% of them used social networking sites to investigate potential employees. In January 2010, the ratio increased to 72%. In addition, at IBM, an IT project manager can find knowledgeable programmers using SmallBlue, a social networking site constructed for IBM staff (Lin et al. 2009). In each of the above situations, trust is one of the most important factors for participants' decision making (Golbeck and Hendler 2006).



[1]http://www.careerbuilder.com/

In social networks, a node represents a participant and links between nodes correspond to physical or online interactions between them. A participant can give a trust value to another based on their interactions. If there is a trust path linking two nonadjacent participants, the source participant can evaluate the trustworthiness of the target one along an existing path based on the *trust transitivity* property (i.e., if $A$ trusts $B$ and $B$ trusts $C$, then $A$ trusts $C$ to some extent) under certain semantic constraints (Jøsang and Pope 2005). The path with trust information linking the source participant and the target one is called a *social trust path* (Hang, Wang, and Singh 2009).

In such a situation, the computation of the value of trust for the target participant requires an understanding of how trust is transitive along a social trust path, which is a critical and challenging problem in OSNs (Guha et al. 2004; Golbeck and Hendler 2006). In the literature, several trust transitivity models have been proposed (Gray et al. 2003; Guha et al. 2004; Golbeck and Hendler 2006; Quercia, Hailes, and Capra 2007; Walter, Battiston, and Schweitzer 2008), but they have the following drawbacks.

Firstly, as illustrated in social psychology (Adler 2001; Lichtenstein and Slovic 2006; Miller, Perlman, and Brehm 2007), the *social relationships* between participants (e.g., the one between an employer and an employee), the *recommendation roles* of participants (e.g., a supervisor as a referee in his postgraduate's job application) and the *preference similarity* between participants (e.g., whether both of them like to play badminton) have significant influence on trust transitivity. However, to the best of our knowledge, these impact factors are not fully considered by existing trust transitivity models. Secondly, a source participant may have different criteria in evaluating the trustworthiness of the target participant (Mansell and Collins 2005), impacting on trust transitivity results. However, the specification of evaluation criteria is not supported by any existing method. Finally, trust transitivity formalized in existing models does not follow the nature of trust decay illustrated in social psychology, namely, trust decays slowly in a certain number of early hops (specified by a source participant) from a source participant, and then decays fast until the trust value approaches the minimum (Gimpel et al. 2008; Jøsang, Gary, and Kinateder 2003).

The significance of trust transitivity and the drawbacks of the existing methods motivate us to propose a new trust transitivity model to compute more resonable trust values between two unknown participants in OSNs. Our main contributions are summarized below.

(1) We first propose a complex social network structure that takes *trust*, *social relationships*, *recommendation roles* and *preference similarity* into account. We then propose a novel concept, Quality of Trust Transitivity (QoTT), taking the above impact factors as the attributes to illustrate the ability of a social trust path to guarantee a certain level of quality in trust transitivity.

(2) Based on the properties of trust illustrated in social psychology, we then propose a new Multiple QoTT Constrained Trust Transitivity (MQCTT) model.

(3) We have conducted experiments on several subnetworks extracted from the *Enron* email dataset[2]. Experimental results demonstrate that our trust transitivity model follows both the principles in social psychology and the properties of trust, and thus it computes more resonable trust values than existing methods.

## Related Work

In the literature, existing models can be classified into three categories based on the types of trust transitivity strategies they adopted. These strategies are 1) multiplication strategy, 2) averaging strategy, and 3) confidence-based strategy.

In the *first category*, the trustworthiness of a target participant is computed as the *multiplication* of the trust values between any two adjacent participants along a social trust path. For example, if $A$ trusts $B$ with $T_{AB}$ and $B$ trusts $C$ with $T_{BC}$ ($T_{AB},\ T_{BC} \in [0,1]$), then $A$ trusts $C$ with $T_{AC} = T_{AB} * T_{BC}$. This strategy has been used in many existing models, e.g., (Walter, Battiston, and Schweitzer 2008; Li, Wang, and Lim 2009).

In the *second category*, the trustworthiness of a target participant is computed based on averaging the trust values between any two adjacent participants along a social trust path. i.e., $T_{AC} = (w_i \cdot T_{AB} + w_j \cdot T_{BC})/2$, where $w_i$ and $w_j$ are the weights of $T_{AB}$ and $T_{BC}$ respectively, and $w_i + w_j = 1$. The trust transitivity models in (Gray et al. 2003; Golbeck and Hendler 2006) belong to this category.

In the *third category*, the confidence between participants is considered in trust transitivity, i.e., $T_{AC}$ is calculated based on $T_{AB}$, $T_{BC}$ and the confidence of $A$ on $T_{BC}$ (denoted as $C_A$) . $C_A$ is computed based on the preference similarity between $A$ and $B$, and it is proportional to the latter. This strategy has been adopted in (Guha et al. 2004; Kuter and Golbeck 2007).

There are some drawbacks in the above three categories of trust transitivity models. Firstly, they do not follow the nature of trust decay illustrated in social psychology (Jøsang, Gary, and Kinateder 2003; Gimpel et al. 2008). Secondly, social psychology (Christianson and Harbison 1996; Adler 2001) also illustrates that trust is not transitive in all situations. For example, Alice trusts Bob (*a football player*) in *playing soccer* and Bob trusts Tom (*a car mechanic*) in *repairing a car*. In such a situation, Alice may not trust Tom in playing soccer. Namely, participants have different recommendation roles (e.g., *a football player* or *a car mechanic*) in different domains (e.g., *playing soccer* or *repairing a car*), which impact on trust transitivity. But existing methods do not consider this impact factor. Moreover, the *social relationships* between participants have significant influence on trust transitivity (Miller, Perlman, and Brehm 2007). However, they are not considered in existing trust transitivity models either. Finally, a source participant should be able to set certain constraints of the above impact factors as criteria for the trust transitivity in different domains (Mansell and Collins 2005; Wang and Varadharajan 2007). But this is not supported by existing methods.

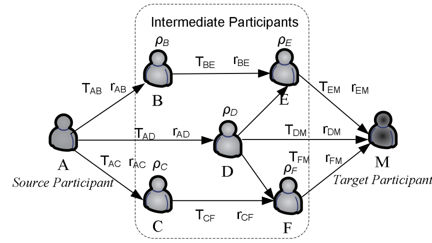


Figure 1: Complex social network

## A Complex Social Network

In this section, we propose a new complex social network structure, as depicted in Fig. 1. It contains the attributes of four impact factors, i.e., *trust*, *social intimacy degree*, *role impact factor* and *preference similarity*.

### Trust

In the literature, many definitions of trust have been proposed addressing different aspects of trust (Golbeck and Hendler 2006; Mansell and Collins 2005). Inspired by these definitions, in the context of this paper, trust between participants in social networks can be defined as "*Trust* is the belief of one in another, based on their interactions, in the extent to which the future action to be performed by the latter will lead to an expected outcome." Let $T_{AB} \in [0,1]$ denote the trust value that $A$ assigns to $B$. $T_{AB} = 0$ indicates that $A$ completely distrusts $B$ while $T_{AB} = 1$ indicates $A$ completely believes $B$'s future action can lead to the expected outcome.

### Social Intimacy Degree

The following principle in social psychology illustrates the impact of the social relationships between participants on trust.

**Principle 1.** A participant can trust participants with whom he/she has more intimate social relationships than those with whom he/she has less intimate social relationships (Miller, Perlman, and Brehm 2007).

Therefore, *Social Intimacy Degree* (SID) between participants should be defined. Let $r_{AB} \in [0,1]$ denote the *Social Intimacy Degree* (SID) between $A$ and $B$. When $r_{AB} = 0$,

[2] http://www.cs.cmu.edu/enron/

$A$ and $B$ have the least intimate social relationship. When $r_{AB} = 1$ they have the most intimate social relationship.

## Role Impact Factor

The recommendation role of a participant also has significant influence on trust.

**Principle 2.** The effective growing knowledge-intensity indeed is a trend towards greater reliance on trust, especially relevant to particular social positions where one's actions weigh heavily on one's social position (Adler 2001).

Let $\rho_A^{D_m} \in [0, 1]$ denote the *Role Impact Factor* (RIF), illustrating the impact of $A$'s recommendation role on trust transitivity in domain $m$ (denoted as $D_m$). When $\rho_A^{D_m} = 1$, $A$ is a domain expert. When $\rho_A^{D_m} = 0$, $A$ has no knowledge in $D_m$.

## Preference Similarity

The following principle in social psychology illustrates the impact of preference similarity on trust.

**Principle 3.** A participant can trust another with whom he/she has higher preference similarity (e.g., both of them like to play badminton) more than those with whom he/she has a lower preference similarity (Luhmann 1979).

Let $S_{AB}^{D_m} \in [0, 1]$ denote the *Preference Similarity* (PS) between $A$ and $B$ in $D_m$. When $S_{AB}^{D_m} = 0$, $A$ and $B$ have no similar preference. When $S_{AB}^{D_m} = 1$, they have the same preference in that domain.

Though it is difficult to build up comprehensive social relationships, recommendation roles and preference similarity hierarchies in all domains, it is feasible to build them up in particular applications. For example, in the work by Mccallum *et al.* (2007), through mining the subjects and contents of emails in *Enron* Corporation[2], the social relationship (e.g., the partnership in funding application) between each email sender and receiver can be discovered and their roles (e.g., a department manager) can be obtained. In addition, at Facebook[3] the preference similarity between two participants can be mined from their profiles (Mislove et al. 2007). Detailed mining methods are out of the scope of this paper.

Since $T$, $r$, $\rho$ and $S$ values are not subjectively specified by a source participant, these factors are called *objective impact factors*.

# Trust Properties and the Quality of Trust Transitivity

In this section, we first analyze trust properties and then propose a novel concept Quality of Trust Transitivity (QoTT).

## The properties of Trust

As illustrated in social psychology, trust has the following properties:

**Property 1: Subjective.** As illustrated in social psychology (Hardin 2002; Mansell and Collins 2005), trust is a subjective phenomenon that is defined by the psychological experiences of the individual who bestows it, reflecting subjective attitudes that affect participants' thinking based on subjective evaluation criteria which can vary in different domains.

[3]http://www.facebook.com/

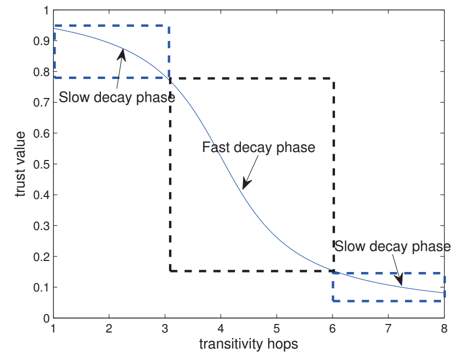


Figure 2: General trust decay with the increase of transitivity hops

**Property 2: Transitive.** Trust can be transitive from one to another with a discount (Christianson and Harbison 1996). In addition, trust transitivity needs certain constraints (Christianson and Harbison 1996; Jøsang and Pope 2005). Namely, if $A$ trusts $B$ in the domain of *teaching C++*, and $B$ trusts $C$ in the domain of *repairing a car*, then the trust cannot be transitive from $A$ to $C$ via $B$ in the domain of *teaching C++*. However, if $A$ also trusts $B$ in *repairing a car* (in the same domain that $B$ trusts $C$), then trust can be transitive from $A$ to $C$ in this domain.

**Property 3: Decay.** In trust transitivity, trust decays with the increase of transitivity hops along a social trust path (Christianson and Harbison 1996). In addition, the general decay is non-linear (Jøsang, Gary, and Kinateder 2003; Mansell and Collins 2005) and can be divided into three phases. ***Phase 1**: (**Slow Decay Phase**)* In this phase, trust decays slowly in transitivity along a social trust path from a source participant within a certain number of hops (e.g., from 1 to 3 hops in Fig. 2). This is because the source participant may consider the familiarity with the trustee to extend no more than a certain number of transitivity hops. ***Phase 2**: (**Fast Decay Phase**)* With the increase of transitivity hops, the trust decay speed increases in trust transitivity until the trust value approaches the minimum (e.g., from 4 to 6 hops in Fig. 2). This is because that in this phase, the trustee is becoming stranger to the source participant than the case in *Phase 1*. ***Phase 3**: (**Slow Decay Phase**)* When the trust value between the source participant and the trustee is approaching the minimum, the trust decay speed changes from fast to slow (e.g., from the $6^{th}$ hop in Fig. 2). This is because in this phase, the trustee has become a stranger to the source participant.

Let $\lambda_1$ denote the number of hops of trust transitivity in Phase 1 (e.g., $\lambda_1 = 3$ in Fig. 2) and $\lambda_2$ denote the number of the hops where trust approaches to zero in Phase 3 (e.g., $\lambda_2 = 8$ in Fig. 2). Their values can be specified by participants based on their own trust evaluation criteria in a certain domain (Jøsang, Gary, and Kinateder 2003; Mansell and Collins 2005). Then even the trust transitivity follows the general trust decay trend along a social trust path, based on different $\lambda_1$ and $\lambda_2$ values specified by the source participants, they can obtain different trust transitivity results of the target along the social trust path.

## Quality of Trust Transitivity (QoTT)

In Service-Oriented Computing (SOC), QoS embodies a set of attributes to illustrate the ability of services to guarantee a certain level of performance (Franken 1996). Similar to the QoS, we propose a novel concept, *Quality of Trust Transitivity*, which in general incorporates any attribute that impacts on trust transitivity.

**Definition 4.** *Quality of Trust Transitivity* (QoTT) is the ability of a social trust path to guarantee a certain level of quality of trust transitivity, taking trust ($T$), social intimacy degree ($r$), role impact factor ($\rho$) and preference similarity ($S$) as attributes.

## QoTT Constraint

Based on *Property 1* of trust, in our model, a source participant can specify multiple end-to-end QoTT constraints for QoTT attributes as the requirements of the Quality of trust transitivity along a social trust path. Let $QoTT^{\mu}$ denote the QoTT constraints for the aggregated QoTT attribute $\mu$ ($\mu \in \{T,\ r,\ \rho,\ S\}$) in a social trust path. In the following, we introduce a method for the aggregation of QoTT attributes in our model.

## The Aggregation Method for QoTT Attributes

**Trust Aggregation** Since trust is discounted with the increase of transitivity hops (Christianson and Harbison 1996), if there are $n$ participants $a_1, ..., a_n$ in order in a social trust path (denoted as $p(a_1, ..., a_n)$), the aggregated trust value is calculated by Eq. (1). This strategy has been widely used in the literature as a feasible trust aggregation method (Walter, Battiston, and Schweitzer 2008; Liu, Wang, and Orgun 2010; Liu et al. 2010).

$$T_{p(a_1,...,a_n)} = \prod_{a_i, a_{i+1} \in p(a_1,...,a_n)} T_{a_i\ a_{i+1}} \tag{1}$$

This aggregated trust value of a path will be regarded as a reference together with the social intimacy degree, the role impact factor and the preference similarity to illustrate the quality of trust transitivity.

**Social Intimacy Degree Aggregation** Firstly, social intimacy between participants decays with the increase of the number of hops between them in a social trust path (Levinger 1983). In addition, the intimacy degree decays fast when it is approaching one, and decays slowly when it is approaching zero (Miller, Perlman, and Brehm 2007). Namely, the decay speed of the social intimacy degree is non-linear in social networks. The aggregated $r$ value in path $p(a_1, ..., a_n)$ can be calculated by Eq.(2) whose function image is a *hyperbolic curve*, fitting the characteristic of social intimacy attenuation.

$$r_{p(a_1,...,a_n)} = \prod_{a_i, a_{i+1} \in p(a_1,...,a_n)} r_{a_i\ a_{i+1}} \tag{2}$$

**Role Impact Factor Aggregation** As illustrated in social psychology (Merton 1957), A social role (e.g., a professor in the field of data mining) is the position of an individual in a given society. Therefore in the same society, the role impact factor of a participant *does not decay* with the increase of transitivity hops. Thus, the aggregated $\rho$ value of path $p_{(a_1,...a_n)}$ in domain $m$ can be calculated by Eq. (3).

$$\rho^{D_m}_{p(a_1,...,a_n)} = \frac{\sum_{k=2}^{n-1} \rho_{a_k}}{n-2} \tag{3}$$

**Preference Similarity Aggregation** As illustrated in social psychology (Lichtenstein and Slovic 2006), if two participants have the same preference to an object, they have a high preference similarity which *does not decay* with the increase of the number of transitivity hops. Thus, the aggregated $S$ value of path $p_{(a_1,...a_n)}$ in domain $m$ can be calculated by Eq. (4).

$$S^{D_m}_{p(a_1,...,a_n)} = \frac{\sum_{k=2}^{n-1} \mathcal{S}_{a_k}}{n-2} \tag{4}$$

Then based on our model, a reliable trust transitivity result can be computed along a social trust path, *if and only if* each aggregated QoTT attribute value of the social trust path satisfies the corresponding end-to-end QoTT constraint.

Since the QoTT constraints, $\lambda_1$ and $\lambda_2$ in trust transitivity are subjectively specified by source participant in trust transitivity, these parameters are called *subjective impact parameters*.

# Multiple QoTT Constrained Trust Transitivity (MQCTT) Model

In this section, we propose a novel Multiple QoTT Constrained Trust Transitivity (MQCTT) model, where both subjective impact parameters and objective impact factors are considered.

In a social trust path $p_{(a_1,...,a_n)}$, with the $\lambda_1$ and $\lambda_2$ specified by the source participant $a_1$, we take $a_{j+1}$ (where there are $j$ hops between $a_1$ and $a_{j+1}$ ($j \leq n-1$)) as an example to introduce the calculation of the trust transitivity result $T_{a_1,a_{j+1}}$ by our MQCTT model.

**Step 1 (average trust decay speed):** Based on *Property 3* of trust, trust decays to zero when the number of transitivity hops is greater than $\lambda_2$ ($\lambda_2 > 1$). As depicted in Fig. 3, we draw a *Base Line* that starts from coordinate $(1,\ T_{a_1,a_2})$, which corresponds to the first hop of trust transitivity with the initial trust value $T_{a_1,a_2}$ and ends at $(\lambda_2 + 1,\ 0)$, where the number of trust transitivity hops is greater than $\lambda_2$, leading to the trust value of zero. This line and its slope can illustrate the *average trust decay speed* along $p_{(a_1,...,a_n)}$ in trust transitivity.

**Step 2 (intersection angle $\theta$):** After identifying the average trust decay speed, based on *Property 3* of trust, if $j \leq \lambda_1$, the trust decay speed should be slower than the average

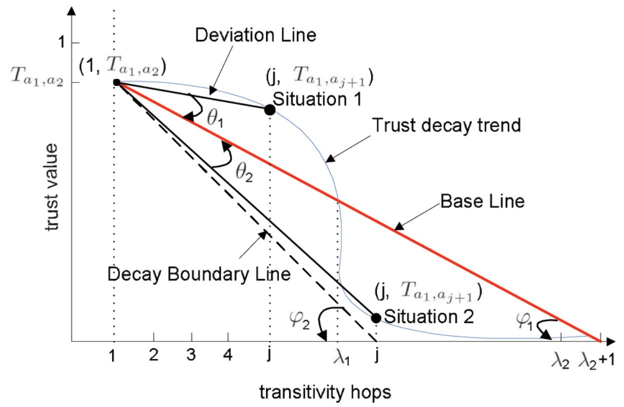


Figure 3: Trust transitivity model

trust decay speed. Therefore, $(j,\ T_{a_1,a_{j+1}})$ should be above the *Base Line* (i.e., *Situation 1* in Fig. 3). If $\lambda_1 < j \leq \lambda_2$ $(j,\ T_{a_1,a_{j+1}})$ should be under the *Base Line* (i.e., *Situation 2* in Fig. 3). A *Deviation Line* that starts from $(1,\ T_{a_1,a_2})$ and ends at $(j,\ T_{a_1,a_{j+1}})$ can be drawn, where an *intersection angle* $\theta$ is formed (i.e., $\theta_1 < 0$ in *Situation 1* or $\theta_2 > 0$ in *Situation 2*). Since the $T_{a_1,a_{j+1}}$ is determined by $\theta$, $\lambda_1$ and $\lambda_2$, in the following steps, we will introduce how to compute the value of $\theta$, and further compute $T_{a_1,a_{j+1}}$ along path $p_{(a_1,\ldots,a_n)}$.

**Step 3 (the scope of $\theta$):** Before computing the value of $\theta$, we first determine the scope of $\theta$. Since trust decays in transitivity from a source participant (Christianson and Harbison 1996), the minimal value of $\theta$ is equal to the *interaction angle* from the *Base Line* to the horizontal axis (i.e., $\varphi_1$ in Fig. 3), which can be calculated by Eq. (5). In addition, based on *Property 3* of trust, if and only if $j > \lambda_2$, $T_{a_1,a_{j+1}}$ decays to zero. We draw a *Decay Boundary Line* that starts from $(1,\ T_{a_1,a_2})$ and ends at $(j,\ 0)$ $j > \lambda_1$ to indicate the trust decay boundary. Then the maximal value of $\theta$ is equal to the interaction angle from *Decay Boundary Line* to the horizontal axis (i.e., $\varphi_2$ in Fig. 3) minus $\varphi_1$, i.e., $\varphi_2 - \varphi_1$, where $\varphi_2$ can be calculated by Eq. (6). Then $\theta \in (\varphi_1, \varphi_2 - \varphi_1)$

$$\varphi_1 = arctan(\frac{T_{a_1,a_2}}{\lambda_2}), \quad \lambda_2 > 1 \text{ and } \varphi_1 \in (0, \frac{\pi}{2}) \tag{5}$$

$$\varphi_2 = arctan(\frac{T_{a_1,a_2}}{j-1}), \quad 1 < j \leq \lambda_2 \text{ and } \varphi_2 \in (0, \frac{\pi}{2}) \tag{6}$$

**Step 4 (logistic function):** As illustrated in *Property 3* of trust, the general trust decay follows the curve plotted in Fig. 2. Therefore, the increase of $\theta$ is non-linear and follows the curve depicted in Fig. 4. In mathematics, the *logistic function* is known to be the most accurate one to model phenomenons possessing non-linear increases with such a trend, and has been widely used in the real-world, e.g., modeling the *non-linear population growth* in ecology, the *non-linear growth of tumors* in medicine and the *nonlinearity of clamp signals* in neural networks (Kingsland 1995). Therefore, to compute an accurate $\theta$ value and further obtain an more reasonable trust transitivity result, we use the *logistic function* as in Eq. (7) to model the increase of $\theta$. The function curve is plotted in Fig. 4.

$$\theta = \begin{cases} [\frac{2*\varphi_1}{1+e^{(\xi-j)}}] - \varphi_1 & \text{for } 1 < j \leq \lambda_1 \\ [\frac{2*(\varphi_2-\varphi_1)}{1+e^{(\xi-j)}}] - (\varphi_2 - \varphi_1) & \text{for } \lambda_1 < j \leq \lambda_2 \end{cases} \tag{7}$$

where $\xi$ is the argument controlling the function curve.

**Step 5 (computing $\theta$ value):** After modeling the increase of $\theta$ by using Eq. (7), it is necessary to calculate the arguments of Eq. (7), and further compute $\theta$ value. From Fig. 4, we can see that $\xi$ is the argument controlling the number of transitivity hops when $\theta = 0$. Then based on *Property 2* of trust, if $0 < j \leq \lambda_1$, then $\xi > \lambda_1$, which ensures $\theta < 0$ (i.e., *Situation 1* in Fig. 3). Otherwise if $\lambda_1 < j \leq \lambda_2$, then $\xi < \lambda_1$, which ensures $\theta > 0$ (i.e., *Situation 2* in Fig. 3). Then $\xi$ can be calculated by Eq. (8) and Eq. (9).

$$\begin{aligned} \tau = r_{p(a_1,\ldots,a_{j+1})} + \rho^{D_m}_{\rho_{p(a_1,\ldots,a_{j+1})}} \\ + S^{D_m}_{p(a_1,\ldots,a_{j+1})} + T_{p(a_1,\ldots,a_{j+1})} \end{aligned} \tag{8}$$

$$\xi = \begin{cases} \lambda_1 + \frac{\tau}{1-\tau} & \text{for } 1 < j \leq \lambda_1 \\ \lambda_1 - \frac{1-\tau}{\tau} & \text{for } \lambda_1 < j \leq \lambda_2 \end{cases} \tag{9}$$

Note that Eq. (8) and Eq. (9) have the following characteristics:

**Characteristic 1:** if $1 < j \leq \lambda_1$ and $\tau \to 0$, then $\xi \to \lambda_1^+$ and thus $\theta \to 0$. In such a situation, the *Deviation Line* tends to coincide with the *Base Line*. Namely, the trust decay speed approaches the average trust decay speed when all QoTT attribute values approach zero.

**Characteristic 2:** If $1 < j \leq \lambda_1$ and $\tau \to 1$, then $\xi \to \infty$ and thus $\theta \to \varphi$. In this situation, the *Deviation Line* tends to be parallel with the horizontal axis. Namely, the trust decay speed approaches zero, when all the QoTT attribute values approach one.

Similarly, we can obtain the same characteristics above when $\lambda_1 < j \leq \lambda_2$, following the principles in social psychology and the properties of trust.

**Step 6 (computing $T_{a_1,a_{j+1}}$ based on $\theta$):** After computing $\theta$ based on Eq. (7) and the slope of *Base Line* (denoted as $k_1$) based on Eq. (10) respectively,

$$k_1 = \frac{T_{a_1,a_2}}{\lambda_2 + 1}, \tag{10}$$

the slope of *Deviation Line* (denoted as $k_2$) can be calculated with Eq. (11).

$$tan(\theta) = (\frac{k_1 - k_2}{1 + k_1 k_2}), \quad \theta \in (-\varphi_1, \varphi_2 - \varphi_1) \tag{11}$$

After obtaining $k_2$, $T_{a_1,a_{j+1}}$ can be calculated by Eq. (12).

$$T_{a_1,a_{j+1}} = T_{a_1,a_2} + k_2 \cdot j, \quad 1 < j \leq \lambda_2 \text{ and } k_2 < 0 \tag{12}$$

$T_{a_1,a_{j+1}}$ is computed based on both objective impact factors (i.e., $T$, $r$, $\rho$ and $S$), and subjective impact parameters

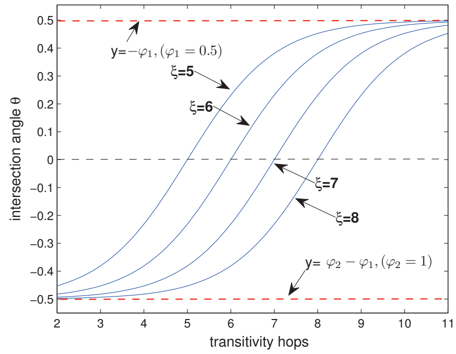


Figure 4: Increase of intersection angle $\theta$

Table 1: Extracted sub-networks

| ID | Max hops | Number of nodes | Number of links |
|---|---|---|---|
| 1 | 4 | 61 | 155 |
| 2 | 4 | 104 | 237 |
| 3 | 5 | 158 | 389 |
| 4 | 5 | 215 | 619 |
| 5 | 6 | 228 | 667 |
| 6 | 6 | 445 | 1418 |
| 7 | 7 | 551 | 3265 |
| 8 | 7 | 750 | 3301 |

(i.e., QoTT constraints, $\lambda_1$ and $\lambda_2$), thus it is different from $T_{p_{(a_1,\ldots,a_{j+1})}}$ which is only one of the above factors impacting on $T_{a_1,a_{j+1}}$ value.

# Experiments

## Experiment Settings

Firstly, in order to evaluate the performance of our proposed MQTTC model, we conduct experiments on sub-networks of different scales and structures, extracted from the *Enron* email dataset[4] which contains 87,474 nodes and 30,0511 links. This dataset has been widely used in the studies of social networks (Mccallum, Wang, and Corrada-Emmanuel 2007; S. Yoo and Moon 2009). We randomly select 8 pairs of source and target participants, and then extract the corresponding 8 sub-networks between them by using an exhaustive search method. Among these sub-networks, the maximal length of a social trust path varies from 4 to 7 hops, following the *small-world* characteristic (Gray et al. 2003)[5]. These sub-networks are listed in Table 1.

Secondly, to compare MQCTT with existing trust transitivity models, we select one model from each of the categories introduced in the section of related work (see Table 2). In addition, we select three domains in our experiments, including (1) *product sales*, (2) *hiring employees* and (3) *making friends*. The values of the subjective impact parameters specified by a source participant are listed in Table 3.

[4]http://www.cs.cmu.edu/enron/

[5]The average path length between any two nodes is about 6.6 hops in a social network

Table 2: Selected trust transitivity models

| Model Number | Category | Strategy | Authors |
|---|---|---|---|
| model 1 | first | multiplication | Walter *et. al* (2008) |
| model 2 | second | average | Golbeck *et. al* (2006) |
| model 3 | third | confidence-based | Guha *et. al* (2004) |

Table 3: Subjective impact parameters of three domains

| Domain (NO.) | $QoTT^T$ | $QoTT^r$ | $QoTT^\rho$ | $QoTT^S$ | $\lambda_1$ | $\lambda_2$ |
|---|---|---|---|---|---|---|
| product sales (1) | 0.1 | 0.05 | 0.05 | 0.05 | 3 | 4 |
| hiring employees (2) | 0.05 | 0.05 | 0.1 | 0.05 | 4 | 5 |
| making friends (3) | 0.05 | 0.05 | 0.05 | 0.1 | 5 | 6 |

Furthermore, the values of $r$, $\rho$ and $S$ can be mined in social networks by using data mining techniques. But this is out of the scope of this paper. Without loss of generality, the values QoTT attributes are randomly generated by using *rand()* in *Matlab*.

Finally, as all trust transitivity models including MQCTT are used to compute the trust value along a social trust path, we compare the most reliable trust transitivity results of all models obtained from the optimal social trust path in a sub-network. The optimal social trust path without QoTT constraints is selected by using the existing optimal algorithm in (Hang, Wang, and Singh 2009), and the path with QoTT constraints in MQCTT is selected by using the optimal algorithm in (Liu, Wang, and Orgun 2010).

All four trust transitivity models are implemented using Matlab R2008a running on an IBM ThinkPad SL500 laptop with an Intel Core 2 Duo T5870 2.00GHz CPU, 3GB RAM, Windows XP SP3 operating system and MySql 5.1.35 database.

## The Performance of MQCTT Model

**Scenario 1: trust transitivity based on different subjective impact parameters** To investigate the performance of the MQCTT model with different subjective impact parameters, we set the same $T$, $r$, $\rho$ and $S$ values in the three domains.

From the experimental results plotted in Fig. 5, we can see that each of the existing trust transitivity models yields the same trust values in the three domains (e.g. S1 in Fig. 5). However, based on *Property 1* of trust, a source participant may have different evaluation criteria in the trust transitivity of different domains, leading to different trust values along the same social trust path. Thus, *existing trust transitivity models neglect this property*.

In contrast, our MQCTT model considers different values of subjective impact parameters specified by the source participant. Therefore, the trust values computed by our MQCTT model are different in the three domains based on the source participant's different trust evaluation criteria (e.g., S2 in Fig. 5), following *Property 1* of trust. In addition, if no social trust path can satisfy the QoTT constraints in the sub-network, or the number of transitivity hops is greater than $\lambda_2$, the source participant will not establish a trust relation with the target participant. Then the trust values of the target participant are equal to zero (e.g., $T = 0$ in S3 in

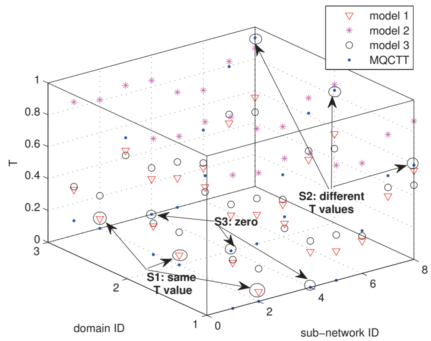


Figure 5: Trust values computed based on different subjective impact parameters

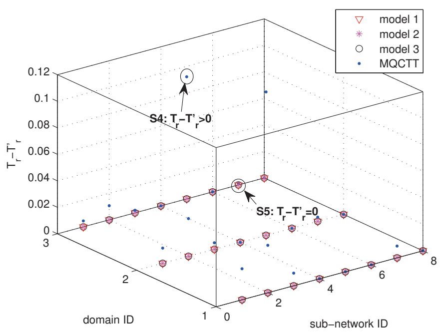


Figure 6: The results of $T_r - T'_r$

Fig. 5). This follows *Properties 2 and 3* of trust. However, existing methods neglect these properties.

**Scenario 2: trust transitivity with different social relationships** To investigate the performance of all models in trust transitivity with different social relationships, $r$ value is decreased to $r' = r/1.5$, and the rest of the QoTT attributes have the same values with those in scenario 1.

Fig. 6 plots the trust transitivity results computed based on $r$ (denoted as $T_r$) minus those computed based on $r'$ (denoted as $T'_r$), i.e., $T_r - T'_r$. We can see that in some cases, $T_r - T'_r > 0$ in the MQCTT model (e.g., S4 in Fig. 6). Namely, the trust value computed by our MQCTT model decreases with the decrease of $r$ value when the social trust path satisfies QoTT constraints, which follows *Principle 1*. In contrast, the trust values computed by each of the three existing trust transitivity models are the same, *neglecting the influence of social relationships*.

In addition, in MQCTT, if the aggregated $r$ and $r'$ values in a path do not satisfy the corresponding QoTT constrains, $T_r = T'_r = 0$ (e.g., S5 in Fig. 6). This follows *Property 3* of trust.

**Scenario 3: trust transitivity with different recommendation roles** To investigate the performance of these models in trust transitivity with different recommendation roles, $\rho^{D3}$ is decreased to $\rho'^{D3} = \rho^{D3}/1.5$. The rest of the QoTT

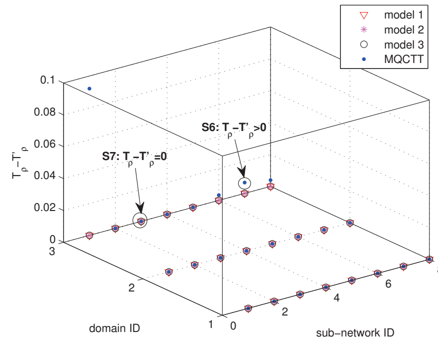


Figure 7: The results of $T_\rho - T'_\rho$

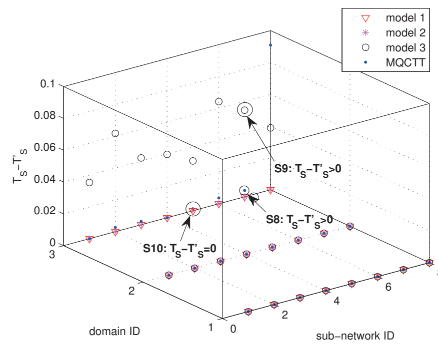


Figure 8: The results of $T_S - T'_S$

attributes have the same values with those in scenario 1.

Fig. 7 plots the trust transitivity results computed based on $\rho^{D3}$ (termed as $T_\rho$) minus those computed based on $\rho'^{D3}$ (termed as $T'_\rho$), i.e., $T_\rho - T'_\rho$. We can see that in some cases in *domain 3*, $T_\rho - T'_\rho > 0$ in our MQCTT model (e.g., S6 in Fig. 7). Namely, the trust value decreases with the decrease of $\rho$ value when the social trust path satisfies the QoTT constraints, which follows *Principle 1*. In contrast, the trust values computed by each of three existing trust transitivity models are the same in each domain, *neglecting the influence of recommendation roles*.

In addition, in MQCTT, if the aggregated $\rho^{D3}$ and $\rho'^{D3}$ value in a path do not satisfy the corresponding QoTT constrains, $T_\rho = T'_\rho = 0$ (e.g., S7 in Fig. 7). This follows *Property 3* of trust.

**Scenario 4: trust transitivity based on different preference similarity** To investigate the performance of these models in trust transitivity with different preference similarity, $S^{D3}$ is decreased to $S'^{D3} = S^{D3}/1.5$. The rest of the QoTT attributes have the same values with those in scenario 1.

Fig. 8 plots the trust transitivity results computed based on $S^{D3}$ (termed as $T_S$) minus those computed based on $\rho'^{D3}$ (termed as $T'_S$), i.e., $T_S - T'_S$. We can see that in some cases in *domain 3*, $T_S - T'_S > 0$ in our MQCTT model (e.g., S8 in Fig. 8). Namely, the trust value computed by our proposed MQCTT model decreases with the decrease of $S$ value when the social trust path satisfies the QoTT constraints, which follows *Principle 1*. In contrast, only the work in (Guha et al. 2004) follows this principle (e.g., S9 in Fig. 8), while other two models neglect the influence of the preference similarity between participants.

In addition, in MQCTT, if the aggregated $S_{D3}$ and $S'^{D3}$ values in a path do not satisfy the corresponding QoTT constrains, $T_S = T'_S = 0$ (e.g., S10 in Fig. 8). This follows *Property 3* of trust. However, the existing methods, including the model in (Guha et al. 2004) do not follow this trust property.

Based on the above experimental results and our analysis in the four scenarios, we can see that our proposed MQCTT model not only follows the principles in social psychology, but also follows the trust properties. Therefore, MQCTT can compute a more resonable trust value of the target participant than existing models.

## Conclusion

In this paper, we have proposed a complex social network structure that takes trust, social relationship, recommendation roles and preference similarity into account. In addition, we proposed a novel general concept of Quality of Trust Transitivity (QoTT) and proposed a novel Multiple QoTT Constrained Trust Transitive (MQCTT) model in complex social networks. Furthermore, we have conducted experiments on a real social network. Experimental results have demonstrated that our MQCTT model follows the principles in social psychology and properties of trust, and thus it computes more resonable trust transitivity results than existing methods.

In our future work, we plan to conduct extensive experiments with some other real social network datasets and develop a visualization tool for trust transitivity analysis in complex social networks.

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