# Grammars

Mark Johnson

joint work with many people, including
Eugene Charniak, Katherine Demuth, Michael Frank,
Sharon Goldwater, Tom Griffiths and Bevan Jones

Macquarie University
Sydney, Australia

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# Why grammars?

- Grammars can define *probability distributions* over infinitely many *trees*
  - "infinite use of finite means"
  - expressive/computational trade-off "sweet spot"
- Why trees?
  - simple model of *hierarchical structure*
  - wide range of applications in language and beyond
- Why probability distributions?
  - *uncertainty is ubiquitous* in perception and learning
  - well-developed mathematics

# Probabilistic languages

- $\mathcal{W}$ is a finite set of *terminal symbols*, a.k.a. the *vocabulary* of the language
  - E.g., $\mathcal{W} = \{\mathrm{likes}, \mathrm{Sam}, \mathrm{Sasha}, \mathrm{thinks}\}$
- A *string* is a *finite sequence* of elements of $\mathcal{W}$
  - E.g., $\mathrm{Sam\ thinks\ Sam\ likes\ Sasha}$
- $\mathcal{W}^\star$ is the *set of all strings* (including the *empty string* $\epsilon$)
- $\mathcal{W}^+$ is the *set of all non-empty strings* (i.e., $\mathcal{W}^+ = \mathcal{W}^\star \setminus \{\epsilon\}$)
- A (formal) *language* is a set of strings (a subset of $\mathcal{W}^\star$)
  - E.g., $L = \{\mathrm{Sam}, \mathrm{Sam\ thinks}, \mathrm{Sasha\ thinks}, \ldots\}$
- A *probabilistic language* is a probability distribution over a language

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# Outline

Context-free grammars

Probabilistic context-free grammars

Learning rule probabilities

Chinese Restaurant Processes

Adaptor grammars

Adaptor grammars for unsupervised word segmentation

Synergies in learning syllables and words

Adaptor grammars for Sesotho morphology

Topic models and learning the referents of words

Learning collocations in LDA topic models

Bayesian inference for adaptor grammars

Conclusion

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# Context-free grammars

- A Context-Free Grammar (CFG) $G = (\mathcal{W}, \mathcal{N}, S, \mathcal{R})$ consists of

  $\mathcal{W}$, a finite set of *terminal symbols*
  $\mathcal{N}$, a finite set of *nonterminal symbols* disjoint from $\mathcal{W}$
  $S \in \mathcal{N}$ is the *start symbol*, and
  $\mathcal{R}$ is a finite set of *rules* or *productions* of the form $A \rightarrow \beta$, where $A \in \mathcal{N}$ and $\beta \in (\mathcal{N} \cup \mathcal{W})^{\star}$

- A *parse tree* generated by CFG $G$ is a finite ordered tree labeled with labels from $\mathcal{N} \cup \mathcal{W}$, where:
  - the *root node* is labeled $S$
  - for each node $n$ labeled with a nonterminal $A \in \mathcal{N}$ there is a rule $A \rightarrow \beta \in \mathcal{R}$ and $n$'s children are labeled $\beta$
  - each node labeled with a terminal has no children

# Example of a CFG parse tree

$$
\begin{aligned}
G &= (\mathcal{W}, \mathcal{N}, \mathrm{S}, \mathcal{R}) \\
\mathcal{W} &= \{\mathrm{barks}, \mathrm{cat}, \mathrm{dog}, \mathrm{the}\} \\
\mathcal{N} &= \{\mathrm{D}, \mathrm{NP}, \mathrm{S}, \mathrm{V}, \mathrm{VP}\} \\
\mathcal{R} &= \left\{ \begin{array}{ccc}
\mathrm{S} \to \mathrm{NP}\ \mathrm{VP} & \mathrm{NP} \to \mathrm{D}\ \mathrm{N} & \mathrm{VP} \to \mathrm{V} \\
\mathrm{D} \to \mathrm{the} & \mathrm{N} \to \mathrm{dog} & \mathrm{V} \to \mathrm{barks} \\
\mathrm{VP} \to \mathrm{V}\ \mathrm{NP} & &
\end{array} \right\}
\end{aligned}
$$

S

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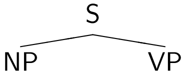
\end{aligned}
$$




$$\Rightarrow \quad \begin{array}{c} \mathrm{S} \\ \mathrm{NP}\ \mathrm{VP} \end{array}$$

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\begin{aligned}
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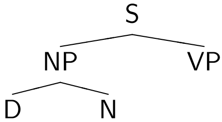


$$
\begin{array}{cc}
 & \mathrm{S} \\
\Rightarrow & \mathrm{NP\ VP} \\
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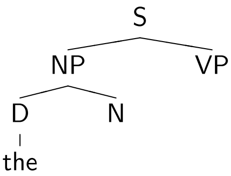


$$
\begin{array}{rc}
 & \mathrm{S} \\
\Rightarrow & \mathrm{NP\ VP} \\
\Rightarrow & \mathrm{D\ N\ VP} \\
\Rightarrow & \mathrm{the\ N\ VP}
\end{array}
$$

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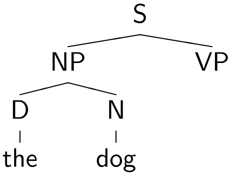


$$
\begin{array}{rc}
 & \mathrm{S} \\
\Rightarrow & \mathrm{NP}\ \mathrm{VP} \\
\Rightarrow & \mathrm{D}\ \mathrm{N}\ \mathrm{VP} \\
\Rightarrow & \text{the}\ \mathrm{N}\ \mathrm{VP} \\
\Rightarrow & \text{the dog}\ \mathrm{VP}
\end{array}
$$

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\begin{aligned}
G &= (\mathcal{W}, \mathcal{N}, \mathrm{S}, \mathcal{R}) \\
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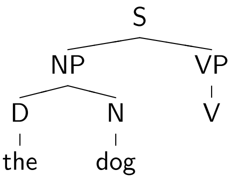


$$
\begin{array}{cc}
 & \mathrm{S} \\
\Rightarrow & \mathrm{NP}\ \mathrm{VP} \\
\Rightarrow & \mathrm{D}\ \mathrm{N}\ \mathrm{VP} \\
\Rightarrow & \text{the}\ \mathrm{N}\ \mathrm{VP} \\
\Rightarrow & \text{the dog}\ \mathrm{VP} \\
\Rightarrow & \text{the dog}\ \mathrm{V}
\end{array}
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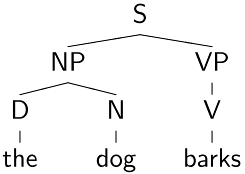


$$
\begin{array}{cc}
 & \mathrm{S} \\
\Rightarrow & \mathrm{NP}\ \mathrm{VP} \\
\Rightarrow & \mathrm{D}\ \mathrm{N}\ \mathrm{VP} \\
\Rightarrow & \text{the N VP} \\
\Rightarrow & \text{the dog VP} \\
\Rightarrow & \text{the dog V} \\
\Rightarrow & \text{the dog barks}
\end{array}
$$

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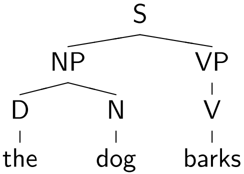


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$$

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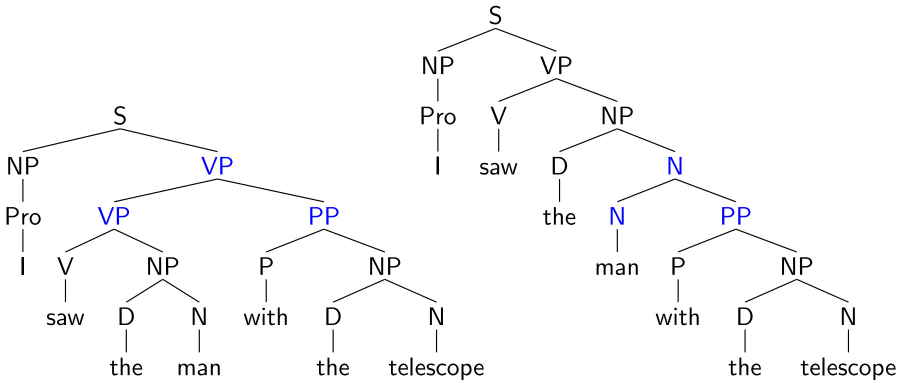
# CFGs can generate structural ambiguity




$$\mathcal{R} = \{\mathrm{VP} \rightarrow \mathrm{V\ NP}, \mathrm{VP} \rightarrow \mathrm{VP\ PP}, \mathrm{NP} \rightarrow \mathrm{D\ N}, \mathrm{N} \rightarrow \mathrm{N\ PP}, \ldots\}$$

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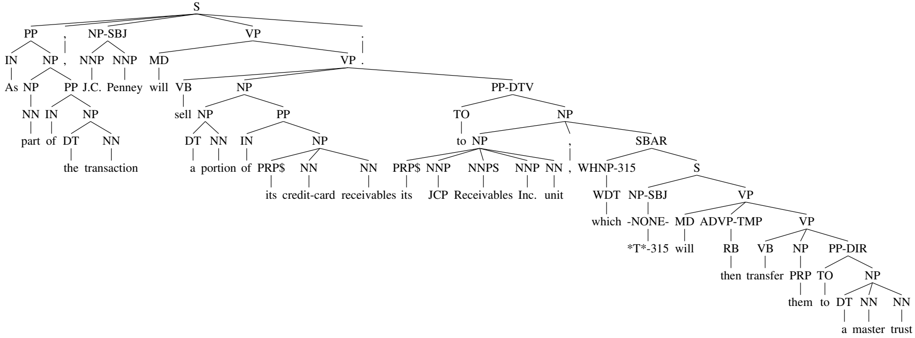
# A small phrase-structure tree from the WSJ




- Identifying phrase structure is a first step in semantic interpretation
- The U Penn Wall Street Journal treebank contains about 55,000 manually-constructed parse trees for about 1,000,000 words of English
- Most modern statistical parsing models are trained from treebanks like this

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# Probabilistic context-free grammars

- A *Probabilistic Context Free Grammar* (PCFG) $G = (\mathcal{W}, \mathcal{N}, S, \mathcal{R}, \boldsymbol{\theta})$ is a 5-tuple where:
  - $(\mathcal{W}, \mathcal{N}, S, \mathcal{R})$ is a CFG with *no useless productions or nonterminals*, and
  - $\boldsymbol{\theta}$ is a vector of *production probabilities*, i.e., a function $\mathcal{R} \to [0, 1]$ that satisfies for each $A \in \mathcal{N}$:

$$\sum_{A \to \beta \,\in\, \mathcal{R}(A)} \theta_{A \to \beta} \;=\; 1$$

    where $\mathcal{R}(A) = \{A \to \alpha : A \to \alpha \in \mathcal{R}\}$.

- A production $A \to \alpha$ is *useless* iff there are no derivations of the form $S \Rightarrow^\star \gamma A \delta \Rightarrow \gamma \alpha \delta \Rightarrow^* w$ for any $\gamma, \delta \in (\mathcal{N} \cup \mathcal{W})^\star$ and $w \in \mathcal{W}^\star$.

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# Probability distribution defined by a PCFG

- Intuitive interpretation:
  - the probability of rewriting nonterminal $A$ to $\alpha$ is $\theta_{A \to \alpha}$
  - the probability of a parse tree $t$ is the *product of probabilities of rules used to build the tree*
- For each production $A \to \alpha \in \mathcal{R}$, let $f_{A \to \alpha}(t)$ be *the number of times $A \to \alpha$ is used in tree $t$*.
- A PCFG $G$ defines a probability distribution $\mathrm{P}_G$ on trees $t$:

$$\mathrm{P}_G(t) \;=\; \prod_{r \in \mathcal{R}} \theta_r^{f_r(t)}$$

- This distribution is properly normalized if $\boldsymbol{\theta}$ satisfies suitable constraints.

# Probabilistic context-free grammars

- Probabilistic context-free grammars (PCFGs) define *probability distributions over trees*
- Each *nonterminal node* expands by:
  - choosing a rule expanding that nonterminal, and
  - recursively expanding any nonterminal children it contains
- Probability of tree is *product of probabilities of rules* used to construct it

| *Probability* $\theta_r$ | *Rule* $r$ |
|---|---|
| 1 | S $\rightarrow$ NP VP |
| 0.7 | NP $\rightarrow$ *Sam* |
| 0.3 | NP $\rightarrow$ *Sandy* |
| 1 | VP $\rightarrow$ V NP |
| 0.8 | V $\rightarrow$ *likes* |
| 0.2 | V $\rightarrow$ *hates* |

S

$$\mathrm{P}(\text{Tree}) =$$

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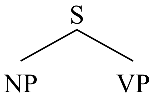


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$$\mathrm{P}(\text{Tree}) = 1 \times$$

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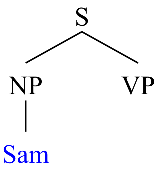


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$$\mathrm{P}(\text{Tree}) = 1 \times 0.7 \times$$

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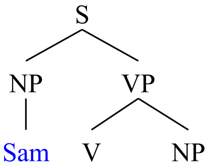


$$\mathrm{P}(\text{Tree}) = 1 \times 0.7 \times 1 \times$$

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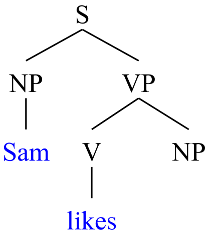


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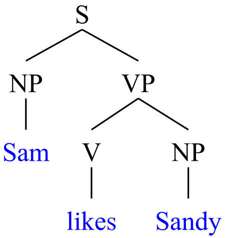


$$\mathrm{P}(\text{Tree}) = 1 \times 0.7 \times 1 \times 0.8 \times 0.3$$

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# Beyond context-free grammars

- Context-free grammars are called "context free" because the expansion of a non-terminal *only depends on the non-terminal's label*
- In a PCFG, each non-terminal expansion is *independent* of the other expansions
  - ⇒ efficient dynamic programming parsing algorithms
  - ⇒ simple learning algorithms
- There is a *hierarchy of context-sensitive grammars*
  - Tree-adjoining grammars, Combinatory categorial grammars, Minimalist grammars
- The *mildly context-sensitive grammars* (MCSGs) have a two-step derivational structure:
  1. Non-deterministically generate a "context-free" *derivation tree*
  2. Deterministically map the derivation tree to a *derived tree*
- (In a CFG, derivation trees and derived trees are the same)

⇒ *MCSGs enjoy all of the nice statistical properties of PCFGs*

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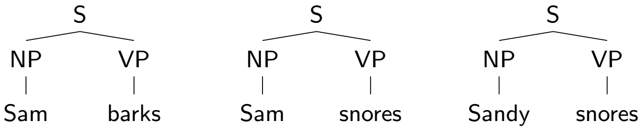
# Maximum likelihood estimation from visible parses

- Each rule expansion is sampled from parent's multinomial

$\Rightarrow$ Maximum Likelihood Estimator (MLE) is rule's *relative frequency*




| Rule $r$ | $n_r$ | $\theta_r$ |
|---|---|---|
| S $\rightarrow$ NP VP | 3 | 1.0 |
| NP $\rightarrow$ Sam | 2 | 0.66 |
| VP $\rightarrow$ barks | 1 | 0.33 |

| Rule $r$ | $n_r$ | $\theta_r$ |
|---|---|---|
| | | |
| NP $\rightarrow$ Sandy | 1 | 0.33 |
| VP $\rightarrow$ snores | 2 | 0.66 |

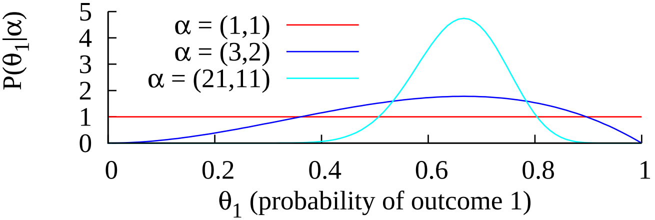
- But MLE is often *overly certain*, especially with sparse data
  - E.g., "accidental zeros" $n_r = 0 \Rightarrow \theta_r = 0$.

# Bayesian estimation from visible parses

- Bayesian estimators estimate a *distribution* over rule probabilities

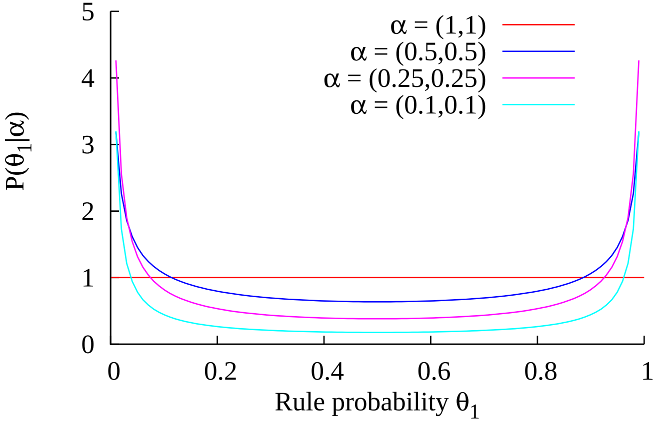
$$\underbrace{\mathrm{P}(\boldsymbol{\theta} \mid \mathbf{n})}_{\text{Posterior}} \propto \underbrace{\mathrm{P}(\mathbf{n} \mid \boldsymbol{\theta})}_{\text{Likelihood}} \underbrace{\mathrm{P}(\boldsymbol{\theta})}_{\text{Prior}}$$

- *Dirichlet distributions* are conjugate priors for multinomials
  - A Dirichlet distribution over $(\theta_1, \ldots, \theta_m)$ is specified by positive parameters $(\alpha_1, \ldots, \alpha_m)$
  - If Prior $= \mathrm{Dir}(\boldsymbol{\alpha})$ then Posterior $= \mathrm{Dir}(\boldsymbol{\alpha} + \mathbf{n})$



# Sparse Dirichlet priors

- As $\alpha \to 0$, Dirichlet distributions become peaked around 0
  "Grammar includes some of these rules, but we don't know which!"



# Estimating rule probabilities from strings alone

- Input: *terminal strings* and *grammar rules*
- Output: *rule probabilities* $\boldsymbol{\theta}$
- In general, no closed-form solution for $\boldsymbol{\theta}$
  - iterative algorithms usually involving *repeatedly reparsing training data*
- *Expectation Maximization* (EM) procedure generalizes visible data ML estimators to hidden data problems
- *Inside-Outside algorithm* is a cubic-time EM algorithm for PCFGs
- Bayesian estimation of $\boldsymbol{\theta}$ via:
  - *Variational Bayes* or
  - *Markov Chain Monte Carlo* (MCMC) methods such as *Gibbs sampling*

# Gibbs sampler for parse trees and rule probabilities

- Input: *terminal strings* $(x_1, \ldots, x_n)$, *grammar rules* and *Dirichlet prior parameters* $\boldsymbol{\alpha}$
- Output: stream of sample *rule probabilities* $\boldsymbol{\theta}$ and *parse trees* $\mathbf{t} = (t_1, \ldots, t_n)$
- Algorithm:

  Assign parse trees to the strings somehow (e.g., randomly)
  Repeat forever:
    Compute rule counts $\mathbf{n}$ from $\mathbf{t}$
    Sample $\boldsymbol{\theta}$ from $\mathrm{Dir}(\boldsymbol{\alpha} + \mathbf{n})$
    For each string $x_i$:
      replace $t_i$ with a tree sampled from $\mathrm{P}(t|x_i, \boldsymbol{\theta})$.

- After *burn-in*, $(\boldsymbol{\theta}, \mathbf{t})$ are distributed according to Bayesian posterior
- Sampling parse tree from $\mathrm{P}(t|x_i, \boldsymbol{\theta})$ involves parsing string $x_i$.

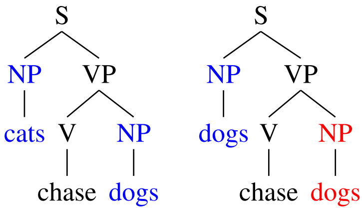
# Collapsed Gibbs samplers

- *Integrate out* rule probabilities $\boldsymbol{\theta}$ to obtain *predictive distribution* $\mathrm{P}(t_i | x_i, \mathbf{t}_{-i})$ of parse $t_i$ for sentence $x_i$ given other parses $\mathbf{t}_{-i}$
- *Collapsed Gibbs sampler*

  For each sentence $x_i$ in training data:

  Replace $t_i$ with a sample from $\mathrm{P}(t | x_i, \mathbf{t}_{-i})$
- A problem: $\mathrm{P}(t_i | x_i, t_{-i})$ is *not a PCFG distribution*

  $\Rightarrow$ no dynamic-programming sampler (AFAIK)

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# Metropolis-Hastings samplers

- Metropolis-Hastings (MH) acceptance-rejection procedure uses samples from a *proposal distribution* to produce samples from a *target distribution*
- When sentence size $\ll$ training data size, $\mathrm{P}(t_i | x_i, \mathbf{t}_{-i})$ is almost a PCFG distribution
  - use a PCFG approximation based on $\mathbf{t}_{-i}$ as *proposal distribution*
  - apply MH to transform proposals to $\mathrm{P}(t_i | x_i, \mathbf{t}_{-i})$
- To construct a Metropolis-Hastings sampler you need to be able to:
  - efficiently sample from proposal distribution
  - calculate ratios of parse probabilities under proposal distribution
  - calculate ratios of parse probabilities under target distribution

# Collapsed Metropolis-within-Gibbs sampler for PCFGs

- Input: *terminal strings* $(x_1, \ldots, x_n)$, *grammar rules* and *Dirichlet prior parameters* $\boldsymbol{\alpha}$
- Output: stream of sample *parse trees* $\mathbf{t} = (t_1, \ldots, t_n)$
- Algorithm:

  Assign parse trees to the strings somehow (e.g., randomly)
  Repeat forever:
    For each sentence $x_i$ in training data:
      Compute rule counts $\mathbf{n}_{-i}$ from $\mathbf{t}_{-i}$
      Compute proposal grammar probabilities $\boldsymbol{\theta}$ from $\mathbf{n}_{-i}$
      Sample a tree $t$ from $\mathrm{P}(t|x_i, \boldsymbol{\theta})$
      Replace $t_i$ with $t$ according to
        Metropolis-Hastings accept-reject formula

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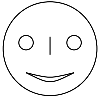
# Example: Learning PCFG rule probabilities (1)

**bazzy** 

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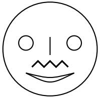
**daxxy** 

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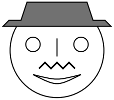
**frobby** 

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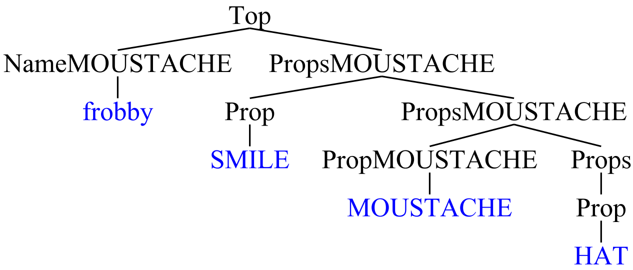
# Example: Learning PCFG rule probabilities (2)

- Input strings: (each string is of form: Name $\text{Property}^+$)

  bazzy SMILE
  daxxy SMILE MOUSTACHE
  frobby SMILE MOUSTACHE HAT

- Rules designed to generate trees as follows:
  - pick a property from property list at random
  - generate name from property




- With *sparse prior* ($\alpha = 10^{-2}$) on $\text{Name}_x \rightarrow y$ rules, learns 1-to-1 mapping between properties and names

# Learning rules (not just their probabilities)

- Input: *terminal strings*
- Output: *grammar rules* and *rule probabilities* $\boldsymbol{\theta}$
- "Generate and test" approach (Carroll and Charniak, Stolcke)

    Guess an initial set of rules
    Repeat:
        re-estimate rule probabilities from strings
        prune low probability rules
        *propose additional potentially useful rules*
- Non-parametric Bayesian methods seem to provide a more systematic approach

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# Non-parameteric Bayesian extensions to PCFGs

- Non-parametric ⇒ no fixed set of parameters
- Two obvious non-parametric extensions to PCFGs:
  - let the set of non-terminals grow unboundedly
    - given an initial grammar with coarse-grained categories, split non-terminals into more refined categories
      $S_{12} \rightarrow NP_7\, VP_4$ instead of $S \rightarrow NP\, VP$.
    - PCFG generalization of "infinite HMM".
  - let the set of rules grow unboundedly ⇒ *adaptor grammars*
    - use a (meta-)grammar to generate potential rules
    - learn subtrees and their probabilities
      i.e., tree substitution grammar, where we learn the fragments as well as their probabilities
- No reason both can't be done at once . . .

# Plan for rest of talk

- Learning structure is hard!
  - Bayesian PCFG estimation works well on toy data, but
  - results are disappointing on real language data
- Strategy: study simpler cases
  - morphological segmentation (e.g., *walking* = *walk*+*ing*)
  - segmenting utterances into words, i.e., learning word pronunciations
  - learning the relationship between words and the objects they refer to
- Idea: extend PCFGs by incorporating non-parametric generalisations of Dirichlet-Multinomials
  - Chinese restaurant processes (Dirichlet processes)
  - Pitman-Yor processes

# Outline

Context-free grammars
Probabilistic context-free grammars
Learning rule probabilities
Chinese Restaurant Processes
Adaptor grammars
Adaptor grammars for unsupervised word segmentation
Synergies in learning syllables and words
Adaptor grammars for Sesotho morphology
Topic models and learning the referents of words
Learning collocations in LDA topic models
Bayesian inference for adaptor grammars
Conclusion

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# Bayesian inference for Dirichlet-multinomials

- Probability of next event with *uniform Dirichlet prior* with mass $\alpha$ over $m$ outcomes and observed data $\mathbf{Z}_{1:n} = (Z_1, \ldots, Z_n)$

$$\mathrm{P}(Z_{n+1} = k \mid \mathbf{Z}_{1:n}, \alpha) \quad \propto \quad n_k(\mathbf{Z}_{1:n}) + \alpha/m$$

  where $n_k(\mathbf{Z}_{1:n})$ is number of times $k$ appears in $\mathbf{Z}_{1:n}$
- Example: Coin ($m = 2$), $\alpha = 1$, $\mathbf{Z}_{1:2} = (\text{heads}, \text{heads})$
  - $\mathrm{P}(Z_3 = \text{heads} \mid \mathbf{Z}_{1:2}, \alpha) \propto 2.5$
  - $\mathrm{P}(Z_3 = \text{tails} \mid \mathbf{Z}_{1:2}, \alpha) \propto 0.5$

# Dirichlet-multinomials with many outcomes


- Predictive probability:

$$\mathrm{P}(Z_{n+1} = k \mid \mathbf{Z}_{1:n}, \alpha) \;\propto\; n_k(\mathbf{Z}_{1:n}) + \alpha/m$$

- Suppose the number of outcomes $m \gg n$. Then:

$$\mathrm{P}(Z_{n+1} = k \mid \mathbf{Z}_{1:n}, \alpha) \;\propto\; \begin{cases} n_k(\mathbf{Z}_{1:n}) & \text{if } n_k(\mathbf{Z}_{1:n}) > 0 \\ \alpha/m & \text{if } n_k(\mathbf{Z}_{1:n}) = 0 \end{cases}$$

- But *most outcomes will be unobserved*, so:

$$\mathrm{P}(Z_{n+1} \notin \mathbf{Z}_{1:n} \mid \mathbf{Z}_{1:n}, \alpha) \;\propto\; \alpha$$

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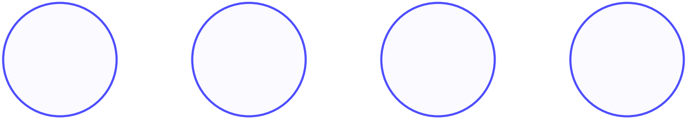
# From Dirichlet-multinomials to Chinese Restaurant Processes

...

- Suppose *number of outcomes is unbounded* but *we* pick the event labels
- If we number event types in order of occurrence $\Rightarrow$ *Chinese Restaurant Process*

$$
\begin{aligned}
Z_1 &= 1 \\
\mathrm{P}(Z_{n+1} = k \mid \mathbf{Z}_{1:n}, \alpha) &\propto \begin{cases} n_k(\mathbf{Z}_{1:n}) & \text{if } k \le m = \max(\mathbf{Z}_{1:n}) \\ \alpha & \text{if } k = m+1 \end{cases}
\end{aligned}
$$

# Chinese Restaurant Process (0)



- Customer $\rightarrow$ table mapping $\mathbf{Z} =$
- $\mathrm{P}(\mathbf{z}) = 1$

- Next customer chooses a table according to:

$$\mathrm{P}(Z_{n+1} = k \mid \mathbf{Z}_{1:n}) \;\propto\; \begin{cases} n_k(\mathbf{Z}_{1:n}) & \text{if } k \leq m = \max(\mathbf{Z}_{1:n}) \\ \alpha & \text{if } k = m + 1 \end{cases}$$

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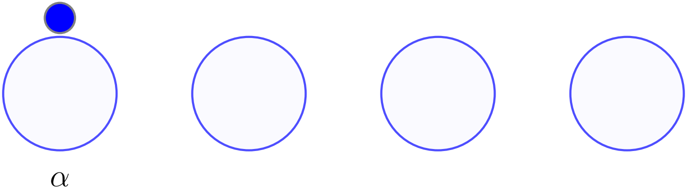
# Chinese Restaurant Process (1)

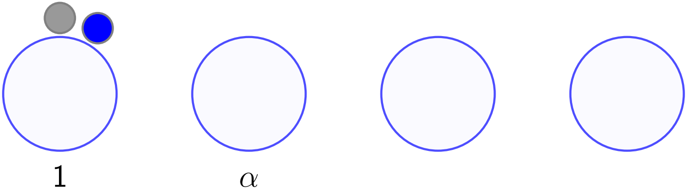



- Customer $\rightarrow$ table mapping $\mathbf{Z} = 1$
- $\mathrm{P}(\mathbf{z}) = \alpha/\alpha$

- Next customer chooses a table according to:

$$\mathrm{P}(Z_{n+1} = k \mid \mathbf{Z}_{1:n}) \propto \begin{cases} n_k(\mathbf{Z}_{1:n}) & \text{if } k \leq m = \max(\mathbf{Z}_{1:n}) \\ \alpha & \text{if } k = m + 1 \end{cases}$$

# Chinese Restaurant Process (2)




- Customer $\rightarrow$ table mapping $\mathbf{Z} = 1, 1$
- $\mathrm{P}(\mathbf{z}) = \alpha/\alpha \times 1/(1+\alpha)$

- Next customer chooses a table according to:

$$\mathrm{P}(Z_{n+1} = k \mid \mathbf{Z}_{1:n}) \;\propto\; \begin{cases} n_k(\mathbf{Z}_{1:n}) & \text{if } k \leq m = \max(\mathbf{Z}_{1:n}) \\ \alpha & \text{if } k = m+1 \end{cases}$$

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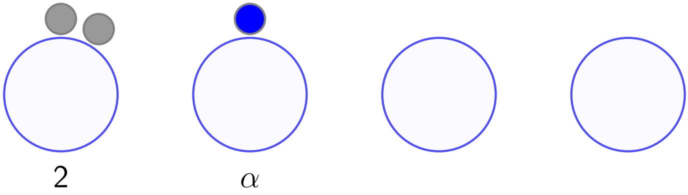
# Chinese Restaurant Process (3)




- Customer $\rightarrow$ table mapping $\mathbf{Z} = 1, 1, 2$
- $\mathrm{P}(\mathbf{z}) = \alpha/\alpha \times 1/(1+\alpha) \times \alpha/(2+\alpha)$

- Next customer chooses a table according to:

$$\mathrm{P}(Z_{n+1} = k \mid \mathbf{Z}_{1:n}) \quad \propto \quad \begin{cases} n_k(\mathbf{Z}_{1:n}) & \text{if } k \leq m = \max(\mathbf{Z}_{1:n}) \\ \alpha & \text{if } k = m + 1 \end{cases}$$

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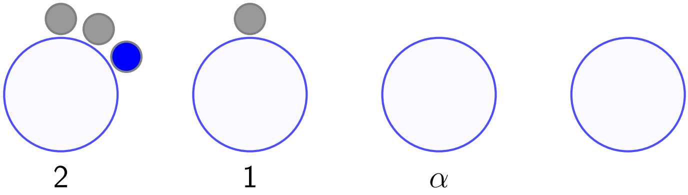
# Chinese Restaurant Process (4)




- Customer $\rightarrow$ table mapping $\mathbf{Z} = 1, 1, 2, 1$
- $\mathrm{P}(\mathbf{z}) = \alpha/\alpha \times 1/(1+\alpha) \times \alpha/(2+\alpha) \times 2/(3+\alpha)$

- Next customer chooses a table according to:

$$\mathrm{P}(Z_{n+1} = k \mid \mathbf{Z}_{1:n}) \;\propto\; \begin{cases} n_k(\mathbf{Z}_{1:n}) & \text{if } k \leq m = \max(\mathbf{Z}_{1:n}) \\ \alpha & \text{if } k = m+1 \end{cases}$$

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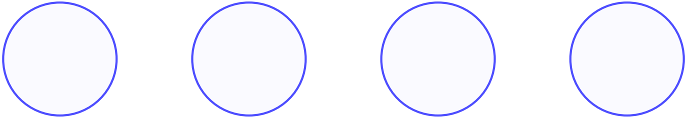
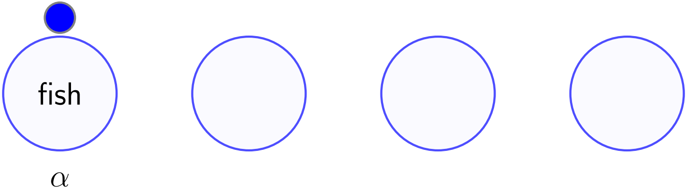
# Labeled Chinese Restaurant Process (0)



- Table → label mapping $\mathbf{Y} =$
- Customer → table mapping $\mathbf{Z} =$
- Output sequence $\mathbf{X} =$
- $\mathrm{P}(\mathbf{X}) = 1$

- *Base distribution* $\mathrm{P}_0(Y)$ generates a *label* $Y_k$ for each table $k$
- All customers sitting at table $k$ (i.e., $Z_i = k$) share label $Y_k$
- Customer $i$ sitting at table $Z_i$ has label $X_i = Y_{Z_i}$

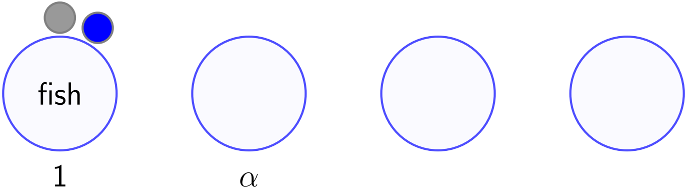
# Labeled Chinese Restaurant Process (1)




- Table $\rightarrow$ label mapping $\mathbf{Y}$ = fish
- Customer $\rightarrow$ table mapping $\mathbf{Z} = 1$
- Output sequence $\mathbf{X}$ = fish
- $\mathrm{P}(\mathbf{X}) = \alpha/\alpha \times \mathrm{P}_0(\text{fish})$

- *Base distribution* $\mathrm{P}_0(Y)$ generates a *label* $Y_k$ for each table $k$
- All customers sitting at table $k$ (i.e., $Z_i = k$) share label $Y_k$
- Customer $i$ sitting at table $Z_i$ has label $X_i = Y_{Z_i}$

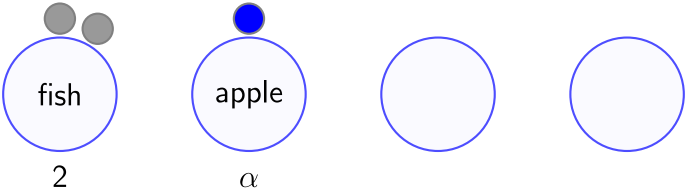
# Labeled Chinese Restaurant Process (2)




- Table → label mapping **Y** = fish
- Customer → table mapping **Z** = 1, 1
- Output sequence **X** = fish,fish
- $\mathrm{P}(\mathbf{X}) = \mathrm{P}_0(\text{fish}) \times 1/(1+\alpha)$

- *Base distribution* $\mathrm{P}_0(Y)$ generates a *label* $Y_k$ for each table $k$
- All customers sitting at table $k$ (i.e., $Z_i = k$) share label $Y_k$
- Customer $i$ sitting at table $Z_i$ has label $X_i = Y_{Z_i}$

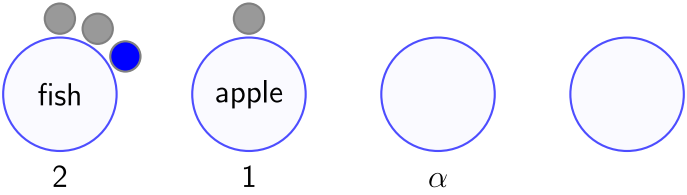
# Labeled Chinese Restaurant Process (3)




- Table $\rightarrow$ label mapping $\mathbf{Y}$ = fish,apple
- Customer $\rightarrow$ table mapping $\mathbf{Z} = 1, 1, 2$
- Output sequence $\mathbf{X}$ = fish,fish,apple
- $\mathrm{P}(\mathbf{X}) = \mathrm{P}_0(\text{fish}) \times 1/(1+\alpha) \times \alpha/(2+\alpha)\mathrm{P}_0(\text{apple})$

- *Base distribution* $\mathrm{P}_0(Y)$ generates a *label* $Y_k$ for each table $k$
- All customers sitting at table $k$ (i.e., $Z_i = k$) share label $Y_k$
- Customer $i$ sitting at table $Z_i$ has label $X_i = Y_{Z_i}$

# Labeled Chinese Restaurant Process (4)




- Table $\rightarrow$ label mapping $\mathbf{Y}$ = fish,apple
- Customer $\rightarrow$ table mapping $\mathbf{Z} = 1, 1, 2$
- Output sequence $\mathbf{X}$ = fish,fish,apple,fish
- $\mathrm{P}(\mathbf{X}) = \mathrm{P}_0(\text{fish}) \times 1/(1+\alpha) \times \alpha/(2+\alpha)\mathrm{P}_0(\text{apple}) \times 2/(3+\alpha)$

- *Base distribution* $\mathrm{P}_0(Y)$ generates a *label* $Y_k$ for each table $k$
- All customers sitting at table $k$ (i.e., $Z_i = k$) share label $Y_k$
- Customer $i$ sitting at table $Z_i$ has label $X_i = Y_{Z_i}$

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# Summary: Chinese Restaurant Processes

- *Chinese Restaurant Processes* (CRPs) generalise Dirichlet-Multinomials to an *unbounded number of outcomes*
  - *concentration parameter* $\alpha$ controls how likely a new outcome is
  - CRPs exhibit a *rich get richer* power-law behaviour
- *Pitman-Yor Processes* (PYPs) generalise CRPs with an additional concentration parameter
  - this parameter specifies the asymptotic power-law behaviour
- *Labeled CRPs* use a *base distribution* to define distributions over arbitrary objects
  - base distribution *"labels the tables"*
  - base distribution can have *infinite support*
  - concentrates mass on a countable subset
  - power-law behaviour $\Rightarrow$ Zipfian distributions

# Nonparametric extensions of PCFGs

- Chinese restaurant processes are a nonparametric extension of Dirichlet-multinomials because the number of states (occupied tables) depends on the data
- Two obvious nonparametric extensions of PCFGs:
  - let the number of nonterminals grow unboundedly
    - *split the nonterminals* of a base grammar
      e.g., $\mathrm{S}_{35} \rightarrow \mathrm{NP}_{27}\ \mathrm{VP}_{17}$

    $\Rightarrow$ infinite PCFG (Finkel et al 2007, Liang et al 2007)
  - let *the number of rules grow unboundedly*
    - "new" rules are compositions of several rules from base grammar
    - equivalent to *caching tree fragments*

    $\Rightarrow$ Adaptor grammars
- No reason both can't be done together ...

# Outline

Context-free grammars

Probabilistic context-free grammars

Learning rule probabilities

Chinese Restaurant Processes

Adaptor grammars

Adaptor grammars for unsupervised word segmentation

Synergies in learning syllables and words

Adaptor grammars for Sesotho morphology

Topic models and learning the referents of words

Learning collocations in LDA topic models

Bayesian inference for adaptor grammars

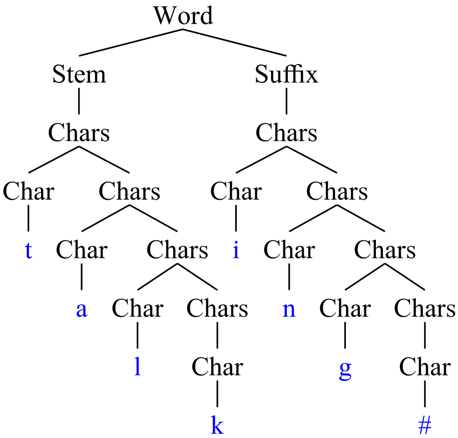
Conclusion

# Adaptor grammars: informal description

- The trees generated by an adaptor grammar are defined by CFG rules as in a CFG
- A subset of the nonterminals are *adapted*
- *Unadapted nonterminals* expand by picking a rule and recursively expanding its children, as in a PCFG
- *Adapted nonterminals* can expand in two ways:
  - by picking a rule and recursively expanding its children, or
  - by generating a previously generated tree (with probability proportional to the number of times previously generated)
- Implemented by having a CRP for each adapted nonterminal
- The CFG rules of the adapted nonterminals determine the *base distributions* of these CRPs

# A CFG for stem-suffix morphology

| | | | | | |
|---|---|---|---|---|---|
| Word | → | Stem Suffix | Chars | → | Char |
| Stem | → | Chars | Chars | → | Char Chars |
| Suffix | → | Chars | Char | → | a \| b \| c \| . . . |



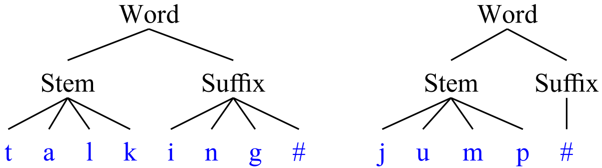

- Grammar's trees can represent any segmentation of words into stems and suffixes

⇒ Can *represent* true segmentation

- But grammar's *units of generalization (PCFG rules) are "too small"* to learn morphemes

# A "CFG" with one rule per possible morpheme

| | | |
|---|---|---|
| Word | $\rightarrow$ | Stem Suffix |
| Stem | $\rightarrow$ | *all possible stems* |
| Suffix | $\rightarrow$ | *all possible suffixes* |

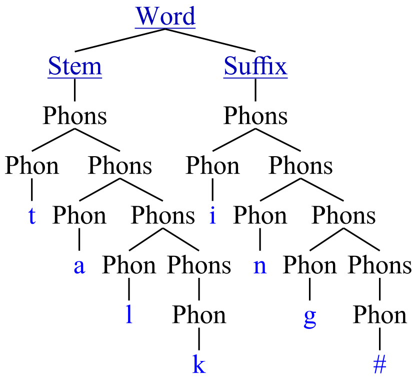



- A rule for each morpheme
  $\Rightarrow$ "PCFG" can represent probability of each morpheme
- *Unbounded number of possible rules, so this is not a PCFG*
  - not a practical problem, as only a finite set of rules could possibly be used in any particular data set

# From PCFGs to Adaptor grammars

- An adaptor grammar is a PCFG where a subset of the nonterminals are *adapted*
- **Adaptor grammar generative process:**
  - to expand an *unadapted nonterminal* $B$: (just as in PCFG)
    - select a *rule* $B \to \beta \in R$ with prob. $\theta_{B \to \beta}$, and recursively expand nonterminals in $\beta$
  - to expand an *adapted nonterminal* $B$:
    - select a *previously generated subtree* $T_B$ with prob. $\propto$ number of times $T_B$ was generated, or
    - select a *rule* $B \to \beta \in R$ with prob. $\propto \alpha_B \, \theta_{B \to \beta}$, and recursively expand nonterminals in $\beta$

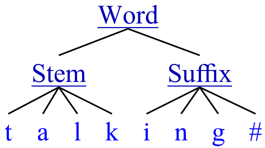
# Adaptor grammar for stem-suffix morphology




Word → Stem Suffix
Stem → Phons
Suffix → Phons
Phons → Phon
Phons → Phon Phons

or in *abbreviated form* with non-adapted nonterminals suppressed

Word → Stem Suffix
Stem → Phon$^+$
Suffix → Phon$^+$

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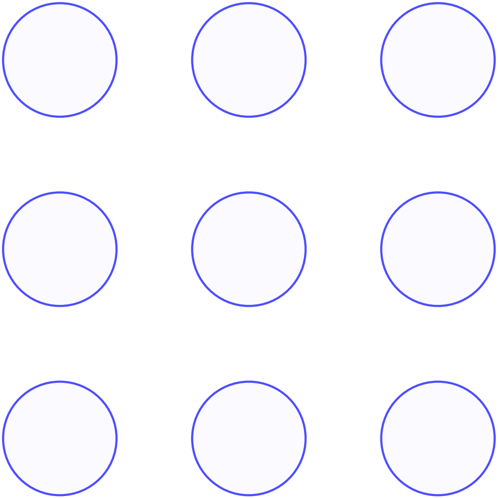
# Adaptor grammar for stem-suffix morphology (0)

Word $\rightarrow$ Stem Suffix

Stem $\rightarrow$ Phoneme$^{+}$

Suffix $\rightarrow$ Phoneme$^{\star}$



Generated words:

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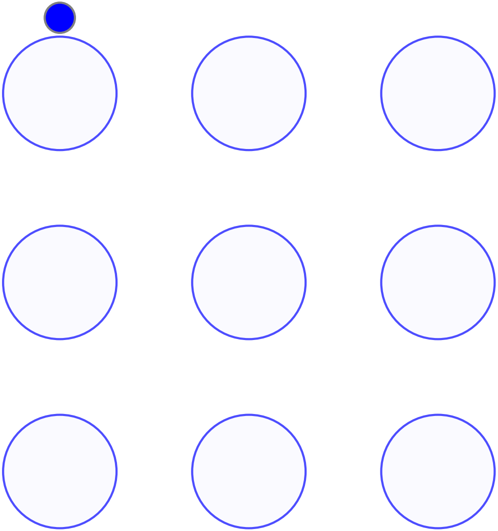
# Adaptor grammar for stem-suffix morphology (1a)

Word → Stem Suffix

Stem → Phoneme$^{+}$

Suffix → Phoneme$^{\star}$



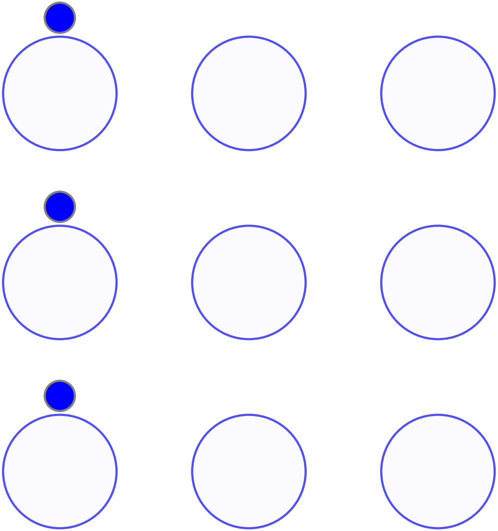
Generated words:

# Adaptor grammar for stem-suffix morphology (1b)

Word $\rightarrow$ Stem Suffix

Stem $\rightarrow$ Phoneme$^{+}$

Suffix $\rightarrow$ Phoneme$^{\star}$



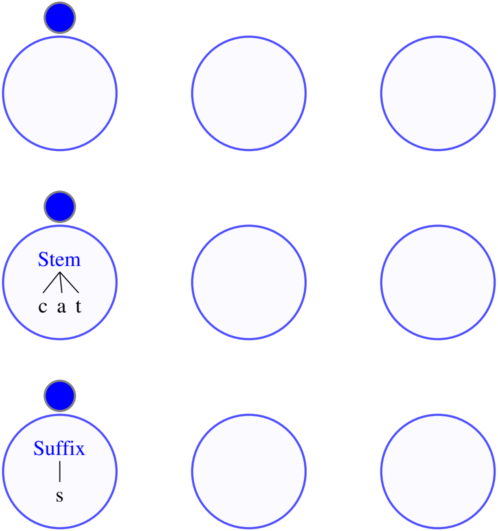
Generated words:

# Adaptor grammar for stem-suffix morphology (1c)

Word → Stem Suffix

Stem → Phoneme$^{+}$

Suffix → Phoneme$^{\star}$



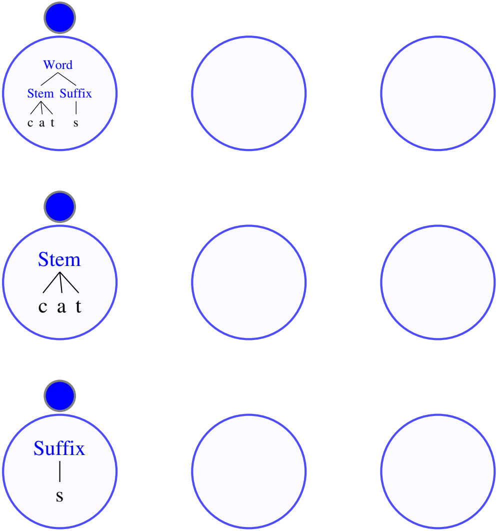

Generated words:

# Adaptor grammar for stem-suffix morphology (1d)

Word → Stem Suffix

Stem → Phoneme$^{+}$

Suffix → Phoneme$^{\star}$




Generated words: cats

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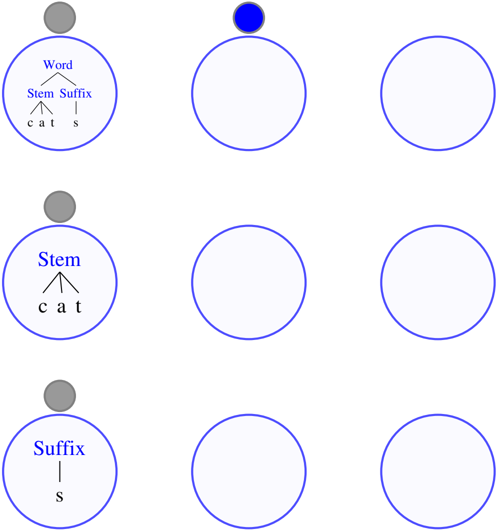
# Adaptor grammar for stem-suffix morphology (2a)

Word → Stem Suffix

Stem → Phoneme$^{+}$

Suffix → Phoneme$^{\star}$



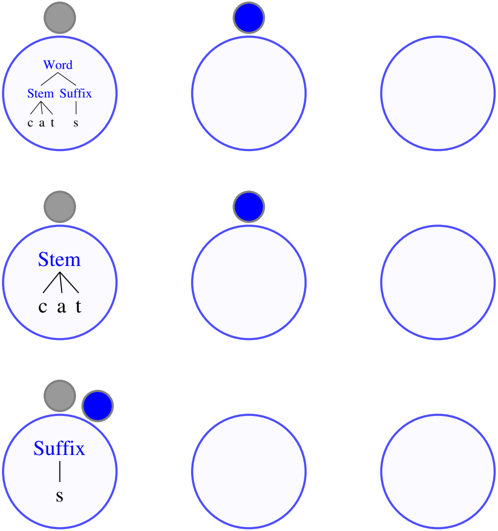

Generated words: cats

# Adaptor grammar for stem-suffix morphology (2b)

Word → Stem Suffix

Stem → Phoneme$^+$

Suffix → Phoneme$^\star$




Generated words: cats

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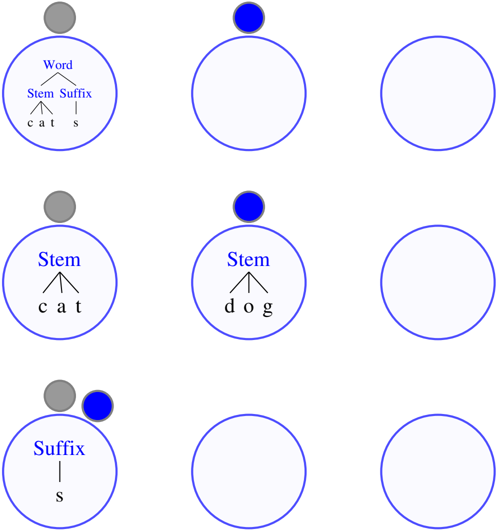
# Adaptor grammar for stem-suffix morphology (2c)

Word → Stem Suffix

Stem → Phoneme$^+$

Suffix → Phoneme$^\star$




Generated words: cats

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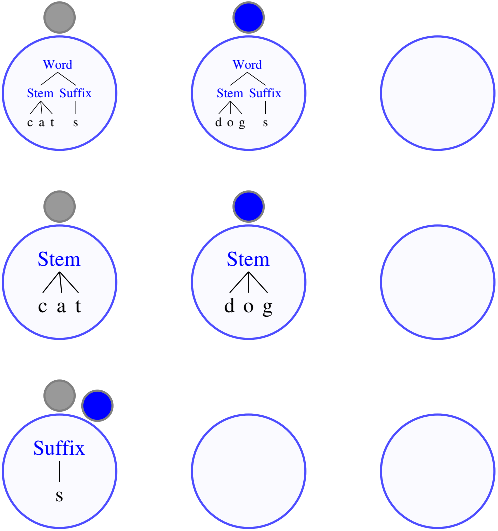
# Adaptor grammar for stem-suffix morphology (2d)

Word → Stem Suffix

Stem → Phoneme$^{+}$

Suffix → Phoneme$^{\star}$




Generated words: cats, dogs

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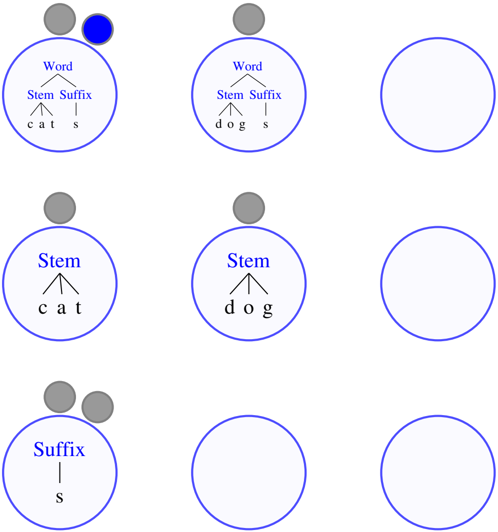
# Adaptor grammar for stem-suffix morphology (3)

Word → Stem Suffix

Stem → Phoneme$^+$

Suffix → Phoneme$^\star$




Generated words: cats, dogs, cats

# Posterior samples from adaptor grammar

| $\alpha = 0.1$ | | $\alpha = 10^{-5}$ | | $\alpha = 10^{-10}$ | | $\alpha = 10^{-15}$ | |
|---|---|---|---|---|---|---|---|
| expect | | expect | | expect | | exp | ect |
| expects | | expect | s | expect | s | exp | ects |
| expected | | expect | ed | expect | ed | exp | ected |
| expect | ing | expect | ing | expect | ing | exp | ecting |
| include | | includ | e | includ | e | includ | e |
| include | s | includ | es | includ | es | includ | es |
| included | | includ | ed | includ | ed | includ | ed |
| including | | includ | ing | includ | ing | includ | ing |
| add | | add | | add | | add | |
| adds | | add | s | add | s | add | s |
| add | ed | add | ed | add | ed | add | ed |
| adding | | add | ing | add | ing | add | ing |
| continue | | continu | e | continu | e | continu | e |
| continue | s | continu | es | continu | es | continu | es |
| continu | ed | continu | ed | continu | ed | continu | ed |
| continuing | | continu | ing | continu | ing | continu | ing |
| repo | | report | | repo | rt | rep | ort |

# Adaptor grammars as generative processes

- The sequence of trees generated by an adaptor grammar are *not* independent
  - it *learns* from the trees it generates
  - if an adapted subtree has been used frequently in the past, it's more likely to be used again
- but the sequence of trees is *exchangable* (important for sampling)
- An *unadapted nonterminal* $A$ expands using $A \to \beta$ with probability $\theta_{A\to\beta}$
- Each adapted nonterminal $A$ is associated with a CRP (or PYP) that caches previously generated subtrees rooted in $A$
- An *adapted nonterminal* $A$ expands:
  - to a subtree $T_A$ rooted in $A$ with probability proportional to the number of times $T_A$ was previously generated
  - using $A \to \beta$ with probability proportional to $\alpha_A \theta_{A\to\beta}$

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# Adaptor grammars as non-parametric PCFGs

- An adaptor grammar *reuses whole previously-generated subtrees* $T_A$ of adapted nonterminals $A$
- This is equivalent to *adding a rule* $A \rightarrow w$ to the grammar, where $w$ is the yield of $T_A$
- If the base CFG generates an *infinite number of trees* $T_A$ for $A$, then the adaptor grammar is *non-parametric*
- But any set of sample parses for a *finite training corpus* only contains a *finite number of number of adapted subtrees*
  - $\Rightarrow$ *sampling methods* (e.g., MCMC) are a natural approach to learning and parsing adaptor grammars
    - in implementation terms, an adaptor grammar is like a PCFG with a *constantly changing set of rules*

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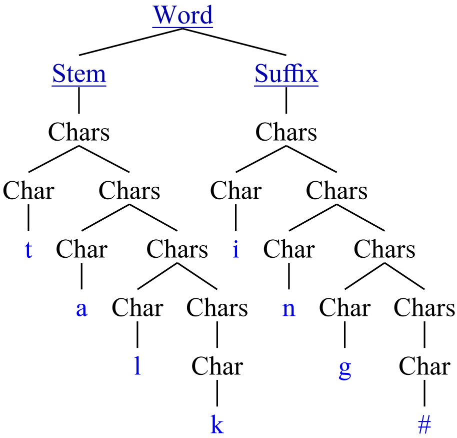
# Properties of adaptor grammars

- Probability of reusing an adapted subtree $T_A$ $\propto$ number of times $T_A$ was previously generated
  - adapted subtrees are *not independent*
    - an adapted subtree can be *more probable* than the rules used to construct it
  - but they are *exchangable* $\Rightarrow$ efficient sampling algorithms
  - "rich get richer" $\Rightarrow$ Zipf power-law distributions
- Each adapted nonterminal is associated with a *Chinese Restaurant Process* or *Pitman-Yor Process*
  - CFG rules define *base distribution* of CRP or PYP
- CRP/PYP parameters (e.g., $\alpha_A$) can themselves be estimated (e.g., slice sampling)

# Bayesian hierarchy inverts grammatical hierarchy

- Grammatically, a Word is composed of a Stem and a Suffix, which are composed of Chars
- To generate a new Word from an Adaptor Grammar:
  - reuse an old Word, or
  - generate a fresh one from the base distribution, i.e., generate a Stem and a Suffix




- *Lower in the tree ⇒ higher in Bayesian hierarchy*

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# Unsupervised word segmentation

- Input: phoneme sequences with *sentence boundaries* (Brent)
- Task: identify *word boundaries*, and hence words

j △ u ▲ w △ ɑ △ n △ t ▲ t △ u ▲ s △ i ▲ ð △ ə ▲ b △ ʊ △ k

"you want to see the book"

- Ignoring phonology and morphology, this involves learning the pronunciations of the lexical items in the language

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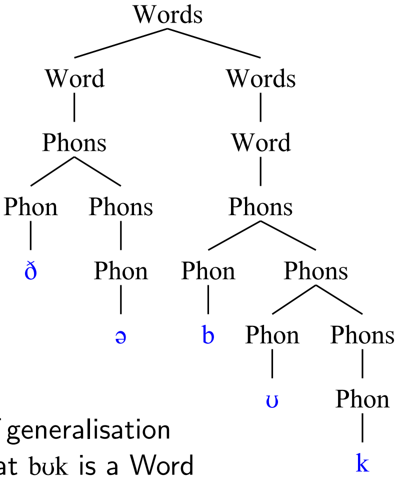
# CFG models of word segmentation

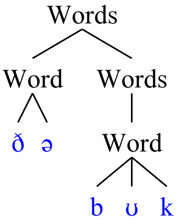
Words → Word
Words → Word Words
Word → Phons
Phons → Phon
Phons → Phon Phons
Phon → *a* | *b* | ...




- CFG trees can *describe* segmentation, but
- PCFGs *can't distinguish* good segmentations from bad ones
  - PCFG rules are *too small* a unit of generalisation
  - need to learn e.g., probability that bʊk is a Word

# Towards non-parametric grammars

Words → Word
Words → Word Words
Word → *all possible phoneme sequences*




- Learn probability Word → b ʊ k
- But *infinitely many possible Word expansions*
  ⇒ this grammar is *not a PCFG*
- Given *fixed training data*, only finitely many useful rules
  ⇒ use data to choose Word rules as well as their probabilities
- An adaptor grammar can do precisely this!

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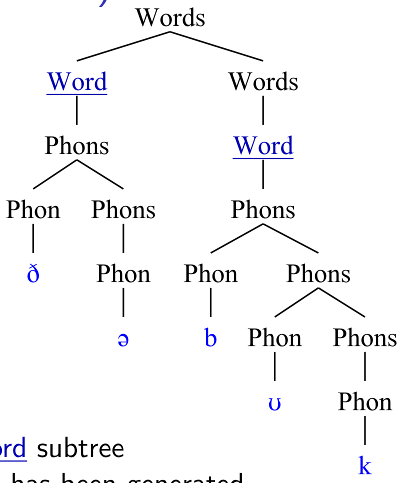
# Unigram adaptor grammar (Brent)

Words → Word
Words → Word Words
Word → Phons
Phons → Phon
Phons → Phon Phons



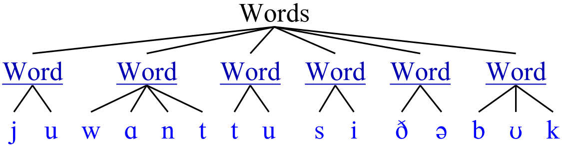

- Word nonterminal is adapted

⇒ To generate a Word:
  - select a previously generated Word subtree with prob. $\propto$ number of times it has been generated
  - expand using Word → Phons rule with prob. $\propto \alpha_{\text{Word}}$ and recursively expand Phons

# Unigram model of word segmentation

- Unigram “bag of words” model (Brent):
  - generate a *dictionary*, i.e., a set of words, where each word is a random sequence of phonemes
    - Bayesian prior prefers smaller dictionaries
  - generate each utterance by choosing each word at random from dictionary
- Brent’s unigram model as an adaptor grammar:

Words $\rightarrow$ Word$^+$
Word $\rightarrow$ Phoneme$^+$




- Accuracy of word segmentation learnt: *56% token f-score* (same as Brent model)
- But we can construct many more word segmentation models using AGs

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# Adaptor grammar learnt from Brent corpus

- **Initial grammar**

| | | | |
|---|---|---|---|
| 1 | Words → Word Words | 1 | Words → Word |
| 1 | Word → Phon | | |
| 1 | Phons → Phon Phons | 1 | Phons → Phon |
| 1 | Phon → *D* | 1 | Phon → *G* |
| 1 | Phon → *A* | 1 | Phon → *E* |

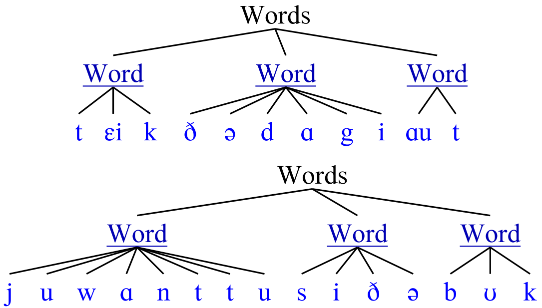
- **A grammar learnt from Brent corpus**

| | | | |
|---|---|---|---|
| 16625 | Words → Word Words | 9791 | Words → Word |
| 1575 | Word → Phons | | |
| 4962 | Phons → Phon Phons | 1575 | Phons → Phon |
| 134 | Phon → *D* | 41 | Phon → *G* |
| 180 | Phon → *A* | 152 | Phon → *E* |
| 460 | Word → (Phons (Phon *y*) (Phons (Phon *u*))) | | |
| 446 | Word → (Phons (Phon *w*) (Phons (Phon *A*) (Phons (Phon *t*))) | | |
| 374 | Word → (Phons (Phon *D*) (Phons (Phon *6*))) | | |
| 372 | Word → (Phons (Phon *&*) (Phons (Phon *n*) (Phons (Phon *d*))) | | |

# Undersegmentation errors with Unigram model

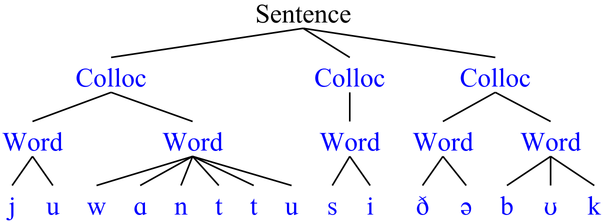
$$\text{Words} \rightarrow \underline{\text{Word}}^{+} \qquad \underline{\text{Word}} \rightarrow \text{Phon}^{+}$$

- Unigram word segmentation model assumes each word is generated independently
- But there are strong inter-word dependencies (collocations)
- Unigram model can only capture such dependencies by analyzing collocations as words (Goldwater 2006)



# Collocations ⇒ Words

Sentence → $\text{Colloc}^+$
Colloc → $\text{Word}^+$
Word → $\text{Phon}^+$




- A Colloc(ation) consists of one or more words
- Both Words and Collocs are adapted (learnt)
- Significantly improves word segmentation accuracy over unigram model (76% f-score; ≈ Goldwater's bigram model)

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# Two hypotheses about language acquisition

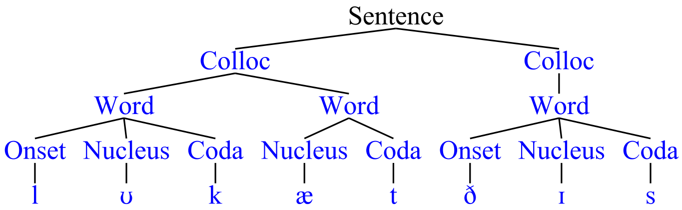
1. Pre-programmed *staged acquisition* of linguistic components
   - Conventional view of *lexical acquisition*, e.g., Kuhl (2004)
     - child first learns the phoneme inventory, which it then uses to learn
     - phonotactic cues for word segmentation, which are used to learn
     - phonological forms of words in the lexicon, . . .
2. *Interactive acquisition* of all linguistic components together
   - corresponds to *joint inference* for all components of language
   - stages in language acquisition might be due to:
     - child's input may contain more information about some components
     - some components of language may be learnable with less data

# Synergies: an advantage of interactive learning

- An *interactive learner* can take advantage of *synergies in acquisition*
  - partial knowledge of component $A$ provides information about component $B$
  - partial knowledge of component $B$ provides information about component $A$
- A staged learner can only take advantage of one of these dependencies
- An interactive or *joint learner* can benefit from a positive feedback cycle between $A$ and $B$
- Are there synergies in *learning how to segment words* and *learning the referents of words*?

# Jointly learning words and syllables

Sentence → Colloc$^+$   Colloc → Word$^+$
Word → Syllable$^{\{1:3\}}$   Syllable → (Onset) Rhyme
Onset → Consonant$^+$   Rhyme → Nucleus (Coda)
Nucleus → Vowel$^+$   Coda → Consonant$^+$




- Rudimentary syllable model (an improved model might do better)
- With 2 Collocation levels, f-score = 84%

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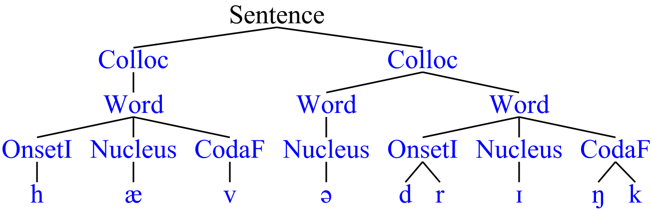
# Distinguishing internal onsets/codas helps

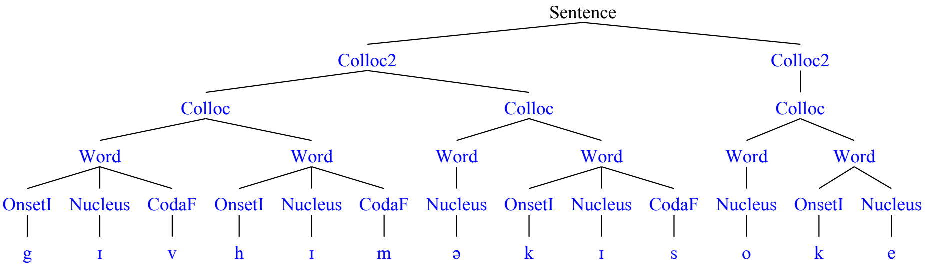
Sentence → Colloc$^+$
Word → SyllableIF
Word → SyllableI Syllable SyllableF
OnsetI → Consonant$^+$
Nucleus → Vowel$^+$

Colloc → Word$^+$
Word → SyllableI SyllableF
SyllableIF → (OnsetI) RhymeF
RhymeF → Nucleus (CodaF)
CodaF → Consonant$^+$




- With 2 Collocation levels, not distinguishing initial/final clusters, f-score = 84%
- With 3 Collocation levels, distinguishing initial/final clusters, f-score = 87%

# Collocations$^2$ ⇒ Words ⇒ Syllables



# Summary of English word segmentation

- Word segmentation accuracy depends on the kinds of generalisations learnt.

| **Generalization** | **Accuracy** |
|---|---|
| words as units (unigram) | 56% |
| + associations between words (collocations) | 76% |
| + syllable structure | 84% |
| + interaction between segmentation and syllable structure | 87% |

- *Synergies in learning words and syllable structure*
  - joint inference permits the learner to *explain away* potentially misleading generalizations

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# The Sesotho corpus

- Sesotho is a Bantu language spoken in southern Africa
- Orthography is (roughly) phonemic
  ⇒ use orthographic forms as broad phonemic representations
- Rich agglutinative morphology (especially in verbs)

  u- e- nk- il- e kae
  SM-OM-take-PERF-IN where
  "You took it from where?"

- The Demuth Sesotho corpus (1992) contains transcripts of child and child-directed speech
- We used a subset of size roughly comparable to Brent corpus of infant-directed speech

| | Brent | Demuth |
|---|---|---|
| utterances | 9,790 | 8,503 |
| word tokens | 33,399 | 30,200 |
| phonemes | 95,809 | 100,113 |

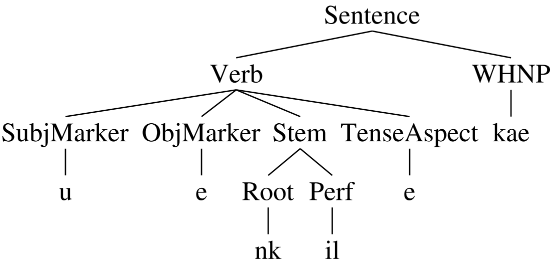
# Sesotho verbs are morphologically complex

u- e- nk- il- e kae
SM-OM-take-PERF-IN where
"You took it from where?"

- Input:

  u ▵ e ▵ n ▵ k ▵ i ▵ l ▵ e ▵ k ▵ a ▵ e

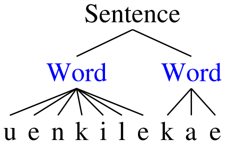
- What I'd like to be able to learn eventually:



# Unigram segmentation grammar – word

u- e- nk- il- e kae
SM-OM-take-PERF-IN where
"You took it from where?"

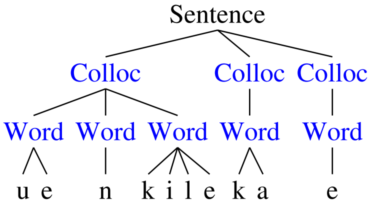
Sentence → Word$^{+}$
Word → Phon$^{+}$




- The word grammar has a word segmentation f-score of 43%
- Lower than 56% f-score on the Brent corpus.
- Sesotho words are longer and more complex.

# Collocation grammar – colloc

Sentence $\rightarrow$ Colloc$^+$
Colloc $\rightarrow$ Word$^+$
Word $\rightarrow$ Phon$^+$

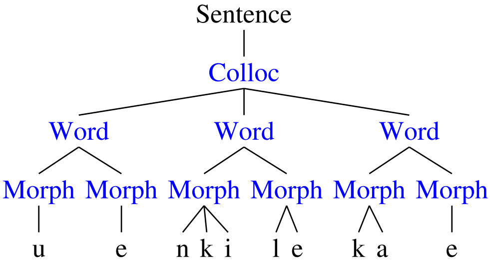



- Learning Collocations improves word segmentation in English; will it help in Sesotho?
- If we treat lower-level units as Words, f-score = 27%
- If we treat upper-level units as Words, f-score = 48%
- English improves by learning dependencies above words, but Sesotho improves by learning generalizations below words

# Adding more levels – colloc2

u- e- nk- il- e kae
SM-OM-take-PERF-IN where
"You took it from where?"

Sentence → Colloc$^+$
Colloc → Word$^+$
Word → Morph$^+$
Morph → Phon$^+$

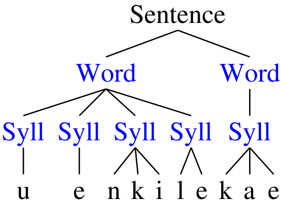



- If two levels are good, maybe three would be better?
- Word segmentation f-score drops to 47%
- Doesn't seem to be much value in adding dependencies above Word level in Sesotho

# Using syllable structure – word-syll

u- e- nk- il- e kae
SM-OM-take-PERF-IN where
"You took it from where?"

$$
\begin{aligned}
&\text{Sentence} \rightarrow \text{Word}^{+}\\
&\underline{\text{Word}} \rightarrow \text{Syll}^{+}\\
&\underline{\text{Syll}} \rightarrow (\text{Onset})\,\text{Nuc}\,(\text{Coda})\\
&\underline{\text{Syll}} \rightarrow \text{SC}\\
&\text{Onset} \rightarrow \text{C}^{+}\\
&\text{Nuc} \rightarrow \text{V}^{+}\\
&\text{Coda} \rightarrow \text{C}^{+}
\end{aligned}
$$

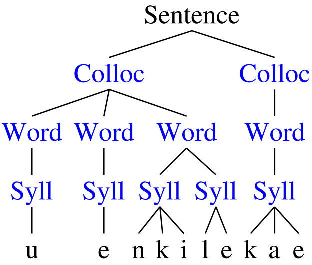



- SC (syllabic consonants) are '*l*', '*m*' '*n*' and '*r*'
- Word segmentation f-score = 50%
- Assuming that words are composed of syllables does improve Sesotho word segmentation

# Using syllable structure – colloc-syll

u- e- nk- il- e kae
SM-OM-take-PERF-IN where
"You took it from where?"

Sentence → Colloc$^+$
Colloc → Word$^+$
Syll → (Onset) Nuc (Coda)
Syll → SC
Onset → C$^+$
Nuc → V$^+$
Coda → C$^+$

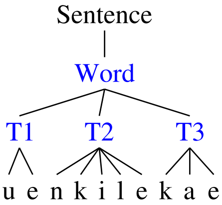



- Word segmentation f-score = 48%
- Additional collocation level doesn't help

# Morpheme positions – word-morph

u- e- nk- il- e kae
SM-OM-take-PERF-IN where
"You took it from where?"

Sentence → Word$^+$
Word → T1 (T2 (T3 (T4 (T5))))
T1 → Phon$^+$
T2 → Phon$^+$
T3 → Phon$^+$
T4 → Phon$^+$
T5 → Phon$^+$

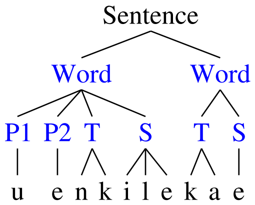



- Each word consists of 1–5 morphemes
- Learn separate morphemes for each position
- Improves word segmentation f-score to 53%

# Building in language-specific information – word-smorph

u- e- nk- il- e kae
SM-OM-take-PERF-IN where
"You took it from where?"

Sentence → Word$^+$
Word → (P1 (P2 (P3))) T (S)
P1 → Phon$^+$
P2 → Phon$^+$
P3 → Phon$^+$
T → Phon$^+$
S → Phon$^+$




- In Sesotho many words consist of a stem T, an optional suffix S and up to 3 prefixes P1, P2 and P3
- Achieves highest f-score = 56%

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Conclusion

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# Prior work: mapping words to referents


- Input to learner:
  - word sequence: *Is that the pig?*
  - objects in nonlinguistic context: DOG, PIG
- Learning objectives:
  - identify utterance topic: PIG
  - identify word-topic mapping: *pig* $\mapsto$ PIG

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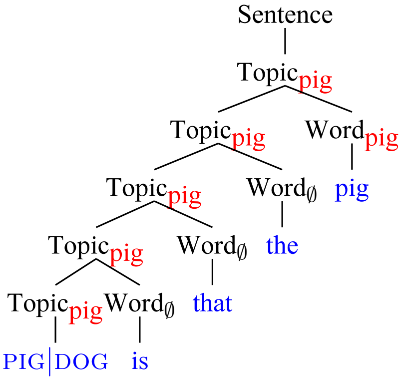
# Frank et al (2009) "topic models" as PCFGs

- Prefix sentences with *possible topic marker*, e.g., PIG|DOG
- PCFG rules *choose a topic* from topic marker and *propagate it through sentence*
- Each word is either generated from sentence topic or null topic ∅

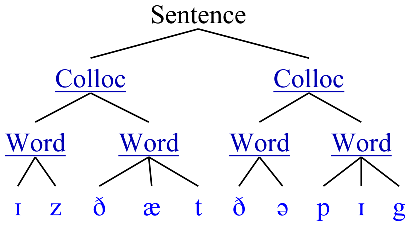



- Grammar can require *at most one topical word per sentence*
- Bayesian inference for PCFG rules and trees corresponds to Bayesian inference for word and sentence topics using topic model (Johnson 2010)

# Word segmentation with adaptor grammars

- Adaptor grammars (AGs) can learn the probability of entire subtrees (as well as rules)
- AGs can express several different word segmentation models
- Learning collocations as well as words significantly improves segmentation accuracy

$$\text{Sentence} \rightarrow \underline{\text{Colloc}}^{+}$$
$$\underline{\text{Colloc}} \rightarrow \underline{\text{Word}}^{+}$$
$$\underline{\text{Word}} \rightarrow \text{Phon}^{+}$$

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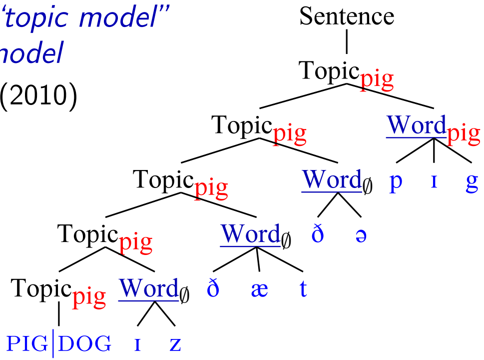
# AGs for joint segmentation and referent-mapping

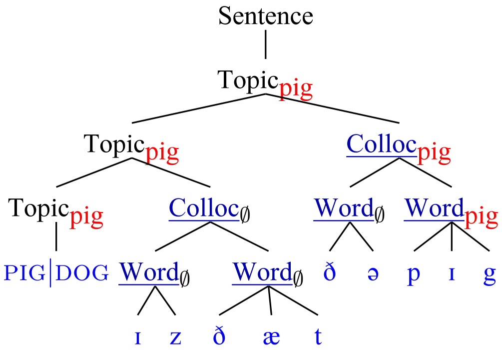
- Combine topic-model PCFG with word segmentation AGs
- Input consists of unsegmented phonemic forms prefixed with possible topics:

PIG|DOG ɪ z ð æ t ð ə p ɪ g

- E.g., combination of *Frank "topic model"* and *unigram segmentation model*
  - equivalent to Jones et al (2010)
- Easy to define *other combinations of topic models and segmentation models*



# Collocation topic model AG




- Collocations are either "topical" or not
- Easy to modify this grammar so
  - at most one topical word per sentence, or
  - at most *one topical word per topical collocation*

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# Experimental set-up

- Input consists of unsegmented phonemic forms prefixed with possible topics:

  PIG|DOG ɪ z ð æ t ð ə p ɪ g

  - Child-directed speech corpus collected by Fernald et al (1993)
  - Objects in visual context annotated by Frank et al (2009)
- Bayesian inference for AGs using MCMC (Johnson et al 2009)
  - Uniform prior on PYP $a$ parameter
  - "Sparse" Gamma$(100, 0.01)$ on PYP $b$ parameter
- For each grammar we ran 8 MCMC chains for 5,000 iterations
  - collected word segmentation and topic assignments at every 10th iteration during last 2,500 iterations
    $\Rightarrow$ 2,000 sample analyses per sentence
  - computed and evaluated the modal (i.e., most frequent) sample analysis of each sentence

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# Does non-linguistic context help segmentation?

| Model | | word segmentation |
| --- | --- | --- |
| **segmentation** | **topics** | **token f-score** |
| unigram | not used | 0.533 |
| unigram | any number | 0.537 |
| unigram | one per sentence | 0.547 |
| collocation | not used | 0.695 |
| collocation | any number | 0.726 |
| collocation | one per sentence | 0.719 |
| collocation | one per collocation | **0.750** |

- Not much improvement with unigram model
  - consistent with results from Jones et al (2010)
- Larger improvement with collocation model
  - most gain with *one topical word per topical collocation* (this constraint cannot be imposed on unigram model)

# Does better segmentation help topic identification?

- Task: identify object (if any) *this sentence* is about

| **Model** | | **sentence referent** | |
|---|---|---|---|
| **segmentation** | **topics** | **accuracy** | **f-score** |
| unigram | not used | 0.709 | 0 |
| unigram | any number | 0.702 | 0.355 |
| unigram | one per sentence | 0.503 | 0.495 |
| collocation | not used | 0.709 | 0 |
| collocation | any number | 0.728 | 0.280 |
| collocation | one per sentence | 0.440 | 0.493 |
| collocation | one per collocation | **0.839** | **0.747** |

- The collocation grammar with *one topical word per topical collocation* is the only model clearly better than baseline

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# Does better segmentation help learning word-to-referent mappings?

- Task: identify *head nouns* of NPs referring to topical objects (e.g. pɪg ↦ PIG in input PIG | DOG ɪ z ð æ t ð ə p ɪ g)

| Model | | topical word |
|---|---|---|
| **segmentation** | **topics** | **f-score** |
| unigram | not used | 0 |
| unigram | any number | 0.149 |
| unigram | one per sentence | 0.147 |
| collocation | not used | 0 |
| collocation | any number | 0.220 |
| collocation | one per sentence | 0.321 |
| collocation | one per collocation | **0.636** |

- The collocation grammar with one topical word per topical collocation is best at identifying head nouns of referring NPs

# Summary of segmentation and word-to-referent mappings

- *Word to object mapping is learnt more accurately when words are segmented more accurately*
  - improving segmentation accuracy improves topic detection and acquisition of topical words
- *Word segmentation accuracy improves when exploiting non-linguistic context information*
  - incorporating word-topic mapping improves segmentation accuracy (at least with collocation grammars)

⇒ *There are synergies a learner can exploit when learning word segmentation and word-object mappings*
  - Caveat: results seem to depend on details of model

- Models limited by ability to simulate "feature-passing" in a PCFG

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# LDA topic models

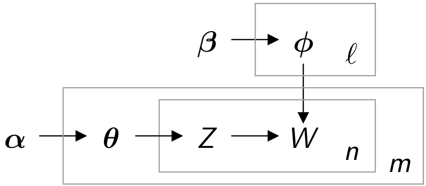
- LDA topic models are *admixture models* of documents
  - topics are assigned to *words* (not sentences or documents)
- An LDA topic model learns:
  - the topics expressed in a document
  - the words characteristic of a topic
- Each topic $i$ is a distribution over words $\boldsymbol{\phi}_i$
- Each document $j$ has a *distribution* $\boldsymbol{\theta}_j$ over topics
- To generate document $j$:
  - for each word position in document:
    - choose a topic $z$ according to $\boldsymbol{\theta}_j$, and then
    - choose a word belonging to that topic according to $\boldsymbol{\phi}_z$
- "Sparse priors" on $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$

  $\Rightarrow$ most documents have few topics

  $\Rightarrow$ most topics have few words

# LDA topic models as Bayes nets

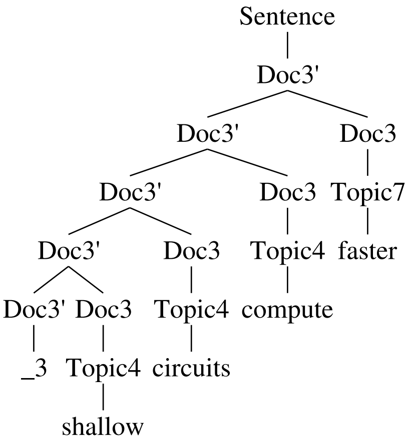
$$\begin{aligned}
\phi_i &\sim \mathrm{Dir}(\beta) & i &= 1, \ldots, \ell = \text{number of topics} \\
\theta_j &\sim \mathrm{Dir}(\alpha) & j &= 1, \ldots, m = \text{number of documents} \\
z_{j,k} &\sim \theta_j & j &= 1, \ldots, m \\
& & k &= 1, \ldots, n = \text{number of words in a document} \\
w_{j,k} &\sim \phi_{z_{j,k}} & j &= 1, \ldots, m \\
& & k &= 1, \ldots, n
\end{aligned}$$



# LDA topic models as PCFGs (1)

- Prefix strings from document $j$ with a *document identifier* "$\_j$"

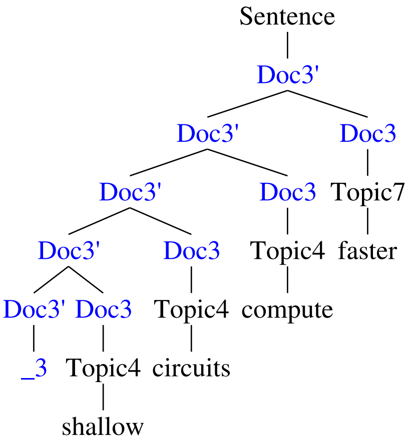
$$
\begin{array}{lll}
\text{Sentence} \rightarrow \text{Doc}'_j & & j \in 1, \ldots, m \\
\text{Doc}'_j \rightarrow \_j & & j \in 1, \ldots, m \\
\text{Doc}'_j \rightarrow \text{Doc}'_j \ \text{Doc}_j & & j \in 1, \ldots, m \\
\text{Doc}_j \rightarrow \text{Topic}_i & & i \in 1, \ldots, \ell \\
 & & j \in 1, \ldots, m \\
\text{Topic}_i \rightarrow w & & i \in 1, \ldots, \ell \\
 & & w \in \mathcal{V}
\end{array}
$$



# LDA topic models as PCFGs (2)

- Spine *propagates document id up through tree*

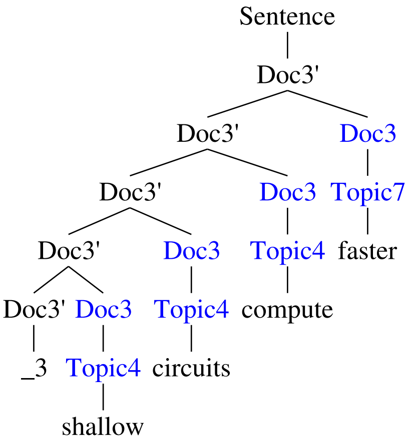
$$
\begin{array}{lll}
\text{Sentence} \rightarrow & \text{Doc}'_j & j \in 1, \ldots, m \\
\text{Doc}'_j \rightarrow & \_j & j \in 1, \ldots, m \\
\text{Doc}'_j \rightarrow & \text{Doc}'_j\ \text{Doc}_j & j \in 1, \ldots, m \\
\text{Doc}_j \rightarrow & \text{Topic}_i & i \in 1, \ldots, \ell \\
 & & j \in 1, \ldots, m \\
\text{Topic}_i \rightarrow & w & i \in 1, \ldots, \ell \\
 & & w \in \mathcal{V}
\end{array}
$$



# LDA topic models as PCFGs (3)

- $\text{Doc}_j \to \text{Topic}_i$ rules map *documents to topics*

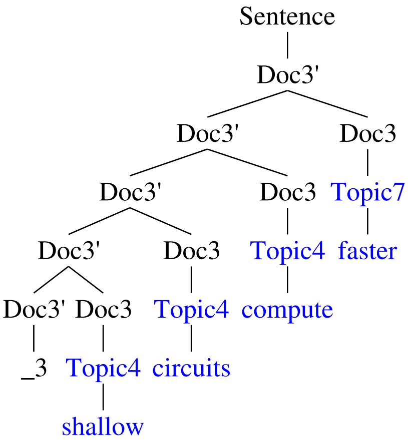
$$
\begin{array}{lll}
\text{Sentence} \to & \text{Doc}'_j & j \in 1, \ldots, m \\
\text{Doc}'_j \to & \_j & j \in 1, \ldots, m \\
\text{Doc}'_j \to & \text{Doc}'_j \ \text{Doc}_j & j \in 1, \ldots, m \\
\text{Doc}_j \to & \text{Topic}_i & i \in 1, \ldots, \ell \\
 & & j \in 1, \ldots, m \\
\text{Topic}_i \to & w & i \in 1, \ldots, \ell \\
 & & w \in \mathcal{V}
\end{array}
$$



# LDA topic models as PCFGs (4)

- $\text{Topic}_i \rightarrow w$ rules map *topics to words*

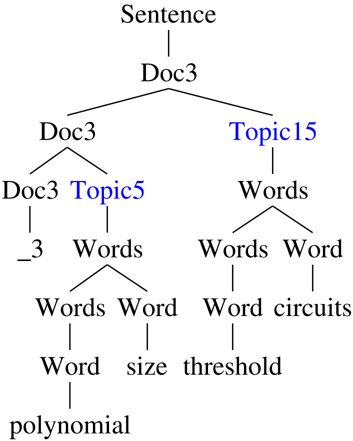
$$
\begin{array}{lll}
\text{Sentence} \rightarrow \text{Doc}'_j & & j \in 1, \ldots, m \\
\text{Doc}'_j \rightarrow \_j & & j \in 1, \ldots, m \\
\text{Doc}'_j \rightarrow \text{Doc}'_j \ \text{Doc}_j & & j \in 1, \ldots, m \\
\text{Doc}_j \rightarrow \text{Topic}_i & & i \in 1, \ldots, \ell \\
 & & j \in 1, \ldots, m \\
\text{Topic}_i \rightarrow w & & i \in 1, \ldots, \ell \\
 & & w \in \mathcal{V}
\end{array}
$$



# Topic model with collocations

- Combines *PCFG topic model* and *segmentation adaptor grammar*

$$
\begin{array}{ll}
\mathrm{Sentence} \to \mathrm{Doc}_j & j \in 1, \ldots, m \\
\mathrm{Doc}_j \to \_j & j \in 1, \ldots, m \\
\mathrm{Doc}_j \to \mathrm{Doc}_j\ \mathrm{Topic}_i & i \in 1, \ldots, \ell; \\
 & j \in 1, \ldots, m \\
\underline{\mathrm{Topic}}_i \to \mathrm{Words} & i \in 1, \ldots, \ell \\
\mathrm{Words} \to \mathrm{Word} & \\
\mathrm{Words} \to \mathrm{Words}\ \mathrm{Word} & \\
\mathrm{Word} \to w & w \in \mathcal{V}
\end{array}
$$



# Finding topical collocations in NIPS abstracts

- Run topical collocation adaptor grammar on NIPS corpus
- Run with $\ell = 20$ topics (i.e., 20 distinct $\text{Topic}_i$ nonterminals)
- Corpus is segmented by punctuation
  - terminal strings are fairly short
  - ⇒ inference is fairly efficient
- Used standard AG implementation
  - Pitman-Yor adaptors
  - sampled Pitman-Yor *a* and *b* parameters
  - flat and "vague Gamma" priors on Pitman-Yor *a* and *b* parameters

# Sample output on NIPS corpus, 20 topics

- Multiword subtrees learned by adaptor grammar:

| | |
|---|---|
| T_0 → gradient descent | T_1 → associative memory |
| T_0 → cost function | T_1 → standard deviation |
| T_0 → fixed point | T_1 → randomly chosen |
| T_0 → learning rates | T_1 → hamming distance |
| T_3 → membrane potential | T_10 → ocular dominance |
| T_3 → action potentials | T_10 → visual field |
| T_3 → visual system | T_10 → nervous system |
| T_3 → primary visual cortex | T_10 → action potential |

- Sample skeletal parses:

  _3 (T_5 polynomial size) (T_15 threshold circuits)
  _4 (T_11 studied) (T_19 pattern recognition algorithms)
  _4 (T_2 feedforward neural network) (T_1 implements)
  _5 (T_11 single) (T_10 ocular dominance stripe) (T_12 low) (T_3 ocularity) (T_12 drift rate)

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# What do we have to learn?

- To learn an adaptor grammar, we need:
  - probabilities of grammar rules
  - adapted subtrees and their probabilities for adapted non-terminals
- If we knew the true parse trees for a training corpus, we could:
  - read off the adapted subtrees from the corpus
  - count rules and adapted subtrees in corpus
  - compute the rule and subtree probabilities from these counts
    - simple computation (smoothed relative frequencies)
- If we aren't given the parse trees:
  - there can be *infinitely many* possible adapted subtrees
  - $\Rightarrow$ can't track the probability of all of them (as in EM)
  - but *sample parses of a finite corpus* only include finitely many
- Sampling-based methods learn the relevant subtrees as well as their weights

# A Gibbs sampler for learning adaptor grammars

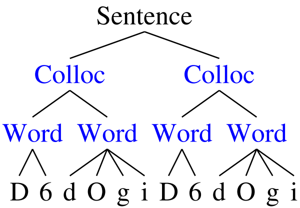
- Gibbs sampling for learning adaptor grammars
  - Assign (random) parse trees to each sentence, and compute rule and subtree counts
  - Repeat forever:
    - pick a sentence (and corresponding parse) at random
    - deduct the counts for the sentence's parse from current rule and subtree counts
    - sample a parse for sentence according to updated grammar
    - add sampled parse's counts to rule and subtree counts
- Sampled parse trees and grammar converges to Bayesian posterior distribution

# Sampling parses from an adaptor grammar

- Sampling a parse tree for a sentence is computationally most demanding part of learning algorithm
- Component-wise Metropolis-within-Gibbs sampler for parse trees:
  - adaptor grammar rules and probabilities *change on the fly*
  - construct PCFG *proposal grammar* from adaptor grammar for previous sentences
  - sample a parse from PCFG proposal grammar
  - use accept/reject to convert samples from proposal PCFG to samples from adaptor grammar
- For particular adaptor grammars, there are often more efficient algorithms

# Details about sampling parses

- Adaptor grammars are *not context-free*
- The probability of a rule (and a subtree) can change within a single sentence
  - breaks standard dynamic programming




- But with moderate or large corpora, the probabilities don't change by much
  - use Metropolis-Hastings accept/reject with a PCFG proposal distribution
- Rules of PCFG proposal grammar $G'(\mathbf{t}_{-j})$ consist of:
  - rules $A \to \beta$ from base PCFG: $\theta'_{A\to\beta} \propto \alpha_A \theta_{A\to\beta}$
  - A rule $A \to \textsc{Yield}(t)$ for each table $t$ in $A$'s restaurant: $\theta'_{A\to\textsc{Yield}(t)} \propto n_t$, the number of customers at table $t$
- Map parses using $G'(\mathbf{t}_{-j})$ back to adaptor grammar parses

# Random vs incremental initialization

- The Gibbs sampler parse trees $\mathbf{t}$ needs to be initialized somehow

  Random initialization: Assign each string $x_i$ a random parse $t_i$ generated by base PCFG

  Incremental initialization: Sample $t_i$ from $\mathrm{P}(t \mid x_i, \mathbf{t}_{1:i-1})$
- Incremental initialization is easy to implement in a Gibbs sampler
- Incremental initialization improves token f-score in all models, especially on simple models

| Model | Random | Incremental |
| --- | --- | --- |
| unigram | 56% | 81% |
| colloc | 76% | 86% |
| colloc-syll | 87% | 89% |

*but see caveats on next slide!*

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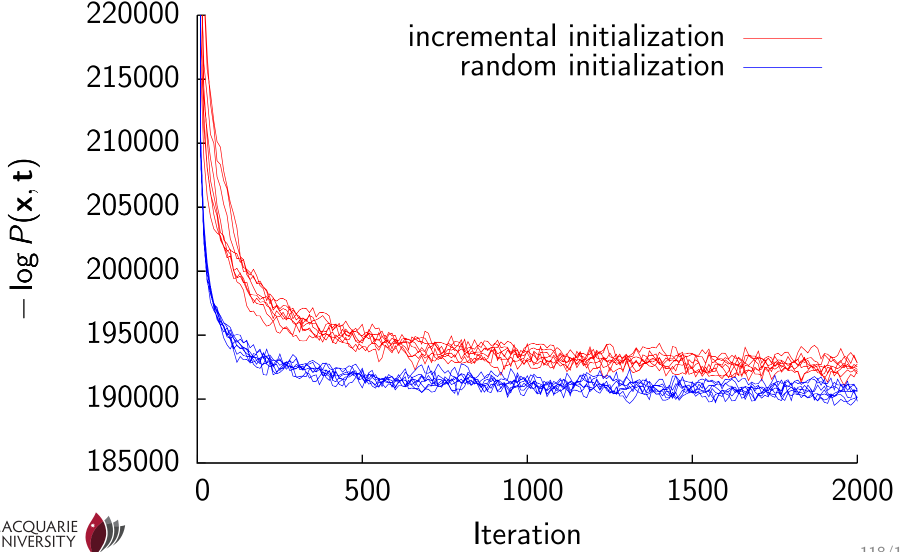
# Incremental initialization produces low-probability parses

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# Why incremental initialization produces low-probability parses

- Incremental initialization produces sample parses **t** with lower probability $\mathrm{P}(\mathbf{t} \mid \mathbf{x})$
- Possible explanation: (Goldwater's 2006 analysis of Brent's model)
  - All the models tend to *undersegment* (i.e., find collocations instead of words)
  - Incremental initialization *greedily searches for common substrings*
  - Shorter strings are more likely to be recurr early than longer ones

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# Table label resampling

- Each adapted non-terminal has a CRP with tables labelled with parses
- "Rich get richer" $\Rightarrow$ resampling a sentence's parse reuses the same cached subtrees
- *Resample table labels* as well sentence parses
  - A table label may be used in many sentence parses
  
  $\Rightarrow$ Resampling a single table label may change the parses of a single sentence
  
  $\Rightarrow$ table label resampling can improve mobility with grammars with a hierarchy of adapted non-terminals
- Essential for grammars with a complex hierarchical structure

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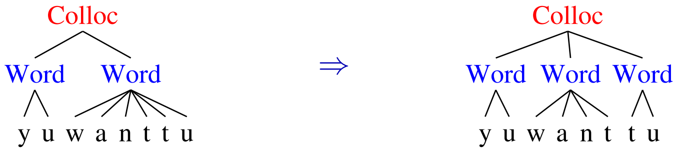
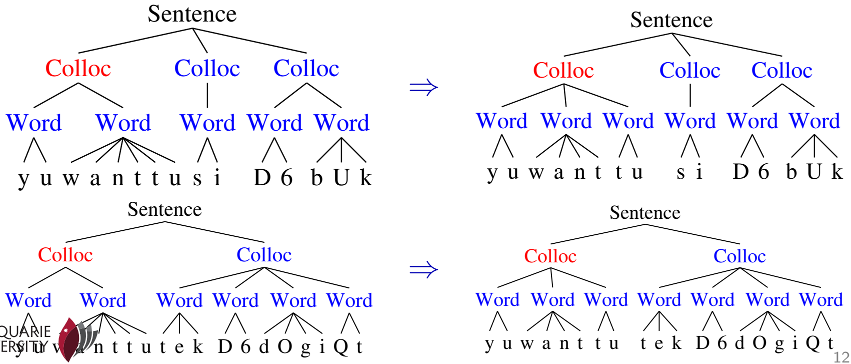
# Table label resampling example

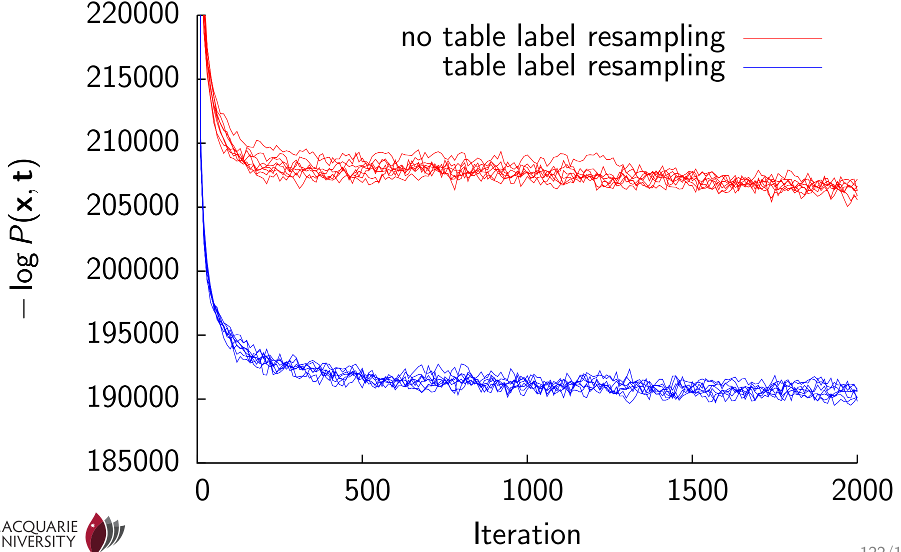
## Label on table in Chinese Restaurant for colloc




## Resulting changes in parse trees



# Table label resampling produces much higher-probability parses



# Summary: learning adaptor grammars

- Unbounded number of possible cached subtrees ⇒ Expectation Maximisation isn't sufficient
- *Gibbs sampler* batch learning algorithm
  - assign every sentence a (random) parse
  - repeatedly cycle through training sentences:
    - withdraw parse (decrement counts) for sentence
    - sample parse for current sentence and update counts
    - Metropolis-Hastings correction

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# Conclusions and future work

- Adaptor grammars can express a variety of useful HDP models
  - generic AG inference code makes it easy to explore a variety of models
- AGs have a variety of applications
  - unsupervised acquisition of morphology
  - unsupervised word segmentation
  - learning word to referent mappings
  - learning collocations in topic models
- *Joint learning* often uses information in the input more effectively than staged learning
- Future work:
  - extend expressive power of AGs (e.g., feature-passing)
  - richer data (e.g., more non-linguistic context)
  - more realistic data (e.g., stress, phonological variation)

# The future of Bayesian models of language acquisition

$$\underbrace{\mathrm{P}(\mathrm{Grammar} \mid \mathrm{Data})}_{\text{Posterior}} \propto \underbrace{\mathrm{P}(\mathrm{Data} \mid \mathrm{Grammar})}_{\text{Likelihood}} \underbrace{\mathrm{P}(\mathrm{Grammar})}_{\text{Prior}}$$

- So far our grammars and priors don't encode much linguistic knowledge, but in principle they can!
  - how do we represent this knowledge?
  - how can we learn efficiently using this knowledge?
- Should permit us to *empirically investigate effects of specific universals on the course of language acquisition*
- My guess: the interaction between innate knowledge and learning will be *richer and more interesting* than either the rationalists or empiricists currently imagine!

# Interested in statistical models, machine learning and computational linguistics?

**Macquarie University** is recruiting
**PhD students** and **post-docs**!

Contact **Mark.Johnson@mq.edu.au** for more information.

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# Context-free grammars

A *context-free grammar* (CFG) consists of:

- a finite set $N$ of *nonterminals*,
- a finite set $W$ of *terminals* disjoint from $N$,
- a finite set $R$ of *rules* $A \to \beta$, where $A \in N$ and $\beta \in (N \cup W)^\star$
- a *start symbol* $S \in N$.

Each $A \in N \cup W$ *generates* a set $\mathcal{T}_A$ of trees.

These are the smallest sets satisfying:

- If $A \in W$ then $\mathcal{T}_A = \{A\}$.
- If $A \in N$ then:

$$\mathcal{T}_A \;=\; \bigcup_{A \to B_1 \ldots B_n \in R_A} \textsc{Tree}_A(\mathcal{T}_{B_1}, \ldots, \mathcal{T}_{B_n})$$

where $R_A \;=\; \{A \to \beta : A \to \beta \in R\}$, and

$$\textsc{Tree}_A(\mathcal{T}_{B_1}, \ldots, \mathcal{T}_{B_n}) \;=\; \left\{ \overset{A}{\overbrace{t_1 \;\ldots\; t_n}} : \begin{array}{l} t_i \in \mathcal{T}_{B_i}, \\ i = 1, \ldots, n \end{array} \right\}$$

The set of trees generated by a CFG is $\mathcal{T}_S$.

# Probabilistic context-free grammars

A *probabilistic context-free grammar* (PCFG) is a CFG and a vector $\boldsymbol{\theta}$, where:

- $\theta_{A \to \beta}$ is the probability of expanding the nonterminal $A$ using the production $A \to \beta$.

It defines distributions $G_A$ over trees $\mathcal{T}_A$ for $A \in N \cup W$:

$$G_A = \begin{cases} \delta_A & \text{if } A \in W \\ \displaystyle\sum_{A \to B_1 \ldots B_n \in R_A} \theta_{A \to B_1 \ldots B_n} \mathrm{TD}_A(G_{B_1}, \ldots, G_{B_n}) & \text{if } A \in N \end{cases}$$

where $\delta_A$ puts all its mass onto the singleton tree $A$, and:

$$\mathrm{TD}_A(G_1, \ldots, G_n)\left(\begin{array}{c} A \\ \diagup \quad \diagdown \\ t_1 \ \ldots \ t_n \end{array}\right) = \prod_{i=1}^{n} G_i(t_i).$$

$\mathrm{TD}_A(G_1, \ldots, G_n)$ is a distribution over $\mathcal{T}_A$ where each subtree $t_i$ is generated independently from $G_i$.

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# DP adaptor grammars

An adaptor grammar $(G, \boldsymbol{\theta}, \boldsymbol{\alpha})$ is a PCFG $(G, \boldsymbol{\theta})$ together with a parameter vector $\boldsymbol{\alpha}$ where for each $A \in N$, $\alpha_A$ is the parameter of the Dirichlet process associated with $A$.

$$\begin{aligned} G_A &\sim \mathrm{DP}(\alpha_A, H_A) \text{ if } \alpha_A > 0 \\ &= H_A \qquad\qquad\quad \text{ if } \alpha_A = 0 \end{aligned}$$

$$H_A = \sum_{A \to B_1 \ldots B_n \in R_A} \theta_{A \to B_1 \ldots B_n} \mathrm{TD}_A(G_{B_1}, \ldots, G_{B_n})$$

The grammar generates the distribution $G_S$.
One Dirichlet Process for each adapted non-terminal $A$ (i.e., $\alpha_A > 0$).

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